Santa Barbara, CA, 100486,
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Colliding plane gravitational waves: Do the focus or not?

I arrived here yesterday and about 24 hours ago I've realized that I was giving the Quantum Gravity Seminar. Well, I only wanted to present you some interesting spacetimes describing the collision of gravitational and electromagnetic waves. So, last night, after the nice Mexican dinner, I've tried to quantize them, but I was not able to complete it. So, I am sorry, you will here a completely classical talk in your Quantum Gravity Seminar.

The ~~general~~ broad problem is the following: Consider the collision and scattering of two plane gravitational waves. General Relativity is non-linear, so, interesting phenomena should arise, not a mere passing through of the waves. We should see non-trivial scattering. Moreover, since gravity is positive, we expect to see some kind of focussing of the waves. The rough question is, how much focusing we should expect.

The conjecture, since the early seventies, associated with the names of Penrose and Szekeres (although Penrose would say, he never made it) is that the focusing is very much substantial and that in any collision of gravitational waves a singularity develops; and the singularity is spacelike and it is compulsory. Even observers who do not really want it, they are forced to see the collision of the waves and experience, subsequenty, a curvature singularity.

The message, the propaganda I will try to pass to you in the next hour or so is that the focusing of the waves is much less than we were thinking so far, in the last 15 years. Well, I will present a spacetime which is an exact solution of the vacuum Einstein equations, it describes the collision of plane gravitational waves, but it develops a timelike, 2-dim. singularity, and only the very dedicated observer, who really want to see it and try hard, will see it.

Everything I will present ~~is~~ in collaboration ~~of~~ with S. Chandrasekhar, of the University of Chicago.

* what kind of waves we are considering? (i) Plane, want to describe the interaction region, two Killing fields the most we can hope for.

* what kinds of plane waves: Ordinary, sandwich, Impulsive. Let me explain. ordinary plane waves, form $g_{ab} = \eta_{ab} + l_a l_b$, l_a = null, direction of propagation. Full theory $l_a l_b$ linear. Hence turn on – wait – turn off,

l^a, sandwich. Idealization, instantaneous duration δ-function amplitude $\Rightarrow$ impulsive. Linear only for propagation in the same direction.

* Simplest situation. Collision of two plane waves, equal magnitudes, impulsive, collide head on.

Lets pose for a moment and think how the spacetime describing such a collision would look like. Four regions. Impulsive, ~~impul~~ shock, impulsive + shock, e.t.c. Trace of gravitational radiation left behind.

* Asymmetrical waves, non-head on collision is just observer dependent, solve only symmetrical, head-on collision.

* Early works, 1970-72, Kahn-Penrose, Szekeres, Bell-Szekeres, strategy, see what are the details of the waves whose collision you would like to describe, evaluate their initial, rather characteristic data on $u=0$, $v=0$, try to solve for the equations in the interaction region – the difficult part – for these initial data.

* About two years ago Chandra and I decided to go into this problem (and six months earlier, Chandra and Ferrari). We wanted to describe the collisions of more involved – always plane-waves, for instance plane gravitational waves coupled with electrodynamic

or hydrodynamic waves. We adopted a new strategy : Look first for solutions in the interaction region, the difficult part, and then try to extend into regions II, III, and IV. Of course, not any solution in region one should be acceptable, and I will explain what ~~I mean~~ the requirements are. But region I has two spacelike commuting Killing fields, so, you do expect to get solutions for the vacuum, the Einstein-Maxwell, and very special Einstein-Perfect fluid cases.

How do you extend the solution, what are the requirements for extendability?

The separating boundaries are null, say $u=0$, $v=0$, u, v null coordinates for region I. I corresponds to $u \geq 0, v \geq 0$. They set $u \to uH(u)$, $v \to vH(v)$. In effect, $u=0$ for II, $v=0$ for III, $u=v=0$ for IV.

- Convince you, good: metric C^0, ~~curvature~~ connection H-function, finite discontinuities, curvature δ-function, we want impulsive waves, O.K.

(i) Demand extendability $\rightleftarrows$ smooth invertible $\exists$ counter examples to both

(ii) Vacuum Einstein equations satisfied II, III. It is not trivial, not a smooth extension.

(α) only vacuum → plane waves

(β) ~~vacuum~~ flat → impulsive waves,

(iii) Vacuum equations on the boundaries, so, everywhere as distributions. No matter fields. It is guarantee by extension and ~~[illegible]~~ a gauge condition we are using, in choosing the null coordinates (u,v). In effect, expect δ-functions in the curvature, it is only in the Weyl part, not in the Ricci part, but $R_{ab}=0$ $R_{ab} \sim (\,)\,H(u)$ ⇒ $R_{ab}=0$ as distribution.

(iv) The extension also gives that the radiating part of the gravitational field is continuous ⇒ conservation of energy?

I am not among the people who communicate with the Gods and then know what will happen in involved situation. So I have to learn what is happening through my equations.

We need exact solutions to ~~[illegible]~~ see what the theory predicts as to what is happening after the collision.

Just a few words about the field equation, only for the vacuum case:

⊛ Two commuting Killing fields,

$$ds^2 = e^{2\Omega}\left[\frac{(d\eta)^2}{1-\eta^2} - \frac{(d\mu)^2}{1-\mu^2}\right] - \sqrt{1-\eta^2}\sqrt{1-\mu^2}\left[\chi\,(dx^2)^2 + \frac{1}{\chi}\,(dx^1 - q_2\,dx^2)^2\right]$$

$\partial/\partial x^1$, $\partial/\partial x^2$ the Killing fields, χ norm, q_2 inner product of the two Killing fields → determine the geometry in the space of the two Killing fields, Ω determines the geometry in the space orthogonal to the two Killing fields, the manifold of fields.

Equation, Ernst equation, $\chi + i q_2 = Z = \frac{1+\mathcal{E}}{1-\mathcal{E}}$,

$$(1-\mathcal{E}\mathcal{E}^*)\left\{\left[(1-\eta^2)\,\mathcal{E}_{,\eta}\right]_{,\eta} - \left[(1-\mu^2)\,\mathcal{E}_{,\mu}\right]_{,\mu}\right\} = -2\,\mathcal{E}^*\left[(1-\eta^2)\,\mathcal{E}_{,\eta}^2 - (1-\mu^2)\,\mathcal{E}_{,\mu}^2\right],$$

independentaly from Ω, then Ω via quadratures.

⊛ $\mathcal{E}$ = real, Killing fields surface orthogonal, parallel polarizations easy solution, like static.

$\mathcal{E}$ complex, different polarizations of the two Killing fields.

Also set $\Psi = \frac{(1-\eta^2)(1-\mu^2)}{\chi}$ = actual squared norm

$\Phi_{,\eta} = \frac{(1-\mu^2)}{\chi^2}\,q_{2,\mu}$, $\Phi_{,\mu} = \frac{(1-\eta^2)}{\chi^2}\,q_{2,\eta}$, twist potential,

$\Psi + i\Phi = Z = \frac{1+E}{1-E}$, it is exactly the ~~Ker~~ same Ernst equation.

⊛ Compare with stationary-axisymmetric, similar but different.

A very simple solution, $\mathcal{E} = p\eta + iq\mu$, $p^2 + q^2 = 1$, two solutions for colliding waves,

one solution for stationary-axisymmetric, it is in fact the Kerr solution.

For colliding waves, solutions ~~studi~~ studied so far, and their physical implications:

(i) Khan–Penrose (1970), $E = y$, surface orthogonal, parallel polarizations,
(ii) Nutku–Halil (1977), $E = py + iq\mu$, different polarization, all are } impulsive
(iii) Szekeres (1972), just plane waves, parallel polarization.

Chandrasekhar + Ferrari (1984), Nutku–Halil in the above formalism.

Conclusions: (i) A spacelike curvature singularity develops, compulsory, a finite distance away to the future

(ii) $v=1$, $u=1$, null singular surfaces. Not curvature, but if you try to extend naturally the world is less dimensional, 2-dimensional.

Geodesics entering from region IV, turn to the right, only those with null $u = $ const or timelike with $P_1 = P_2 = 0$, no motion in the other two directions.

Physical interpretation, if you are wondering around, or running fast but ~~...~~ slightly off course, the second wave will catch you, you will experience the collision.

(iii) Back to the curvature singularity: $s = 1 - u^2 - v^2$ distance away, then

$R^{abcd} R_{abcd} \sim 1/s^7$ Khan–Penrose, Nutku–Halil $\sim 1/s^3$

* Interpretation, non parallel polarizations, the focusing mechanism is less efficient

The feeling, the believe was that the singularity was there. So 2 years ago, when we started with Chandra, we never contested it. We went to see what happens in other situations, retain plane waves (two Killing fields in the interaction region) but have electromagnetic or hydrodynamic waves coupled to gravitational ones. We obtained explicitly some spacetimes — exact solutions Einstein–Maxwell, or Einstein–extremely relativistic fluid — I am not going to discuss them, but as far as singularities, nothing changes. ~~Sp...~~ Curvature inevitable singularity there, and non-parallel polarization simply reduces the rate of blowing up.

Now let me come to the new solution for colliding waves (the pretty new paper was submitted Tuesday morning).

We have an exact solution of the Einstein equations for this problem ~~which suggests~~ in which $u^2+v^2=1$ is not a singularity, it is extendable. Later on, it develops a much milder singularity.

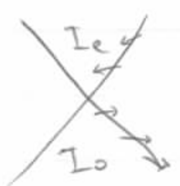
Analytically the solution is simple, $\mathcal{E}=p\eta+iq\mu$, $p^2+q^2=1$, but for $\mathcal{E}$ arising from the norm and the twist of the Killing field. So, just mathematically, for those just interesting in getting solutions of this equations, it is not very interesting, nothing new. Its analysis and physical interpretation are interesting. A few facts about it.

(i) We worked, at $u^2+v^2=1$ no curvature singularity develops. All Weyl scalars of Newman-Penrose formalism are finite $\Rightarrow$ any curvature invariant will be finite.

⊗ We were stuck for about 2-months, is it extendable or not? Well Chandra would not hesitate to go to England for two days to talk with Roger Penrose. It was worth. With some crucial suggestion from ~~th~~ him, we were able to extend, establishing that what was in all previous solutions a curvature singularity, in the present case was a mere coordinate singularity.

⊗ How the extension: (u,v,x^1,x^2), $s=1-u^2-v^2$, $r=u^2-v^2$, from the two Killing fields, $\partial/\partial x^2$ null, $\partial/\partial x^1$ spacelike, take
$\xi = s e^{x_2/q}$, $\zeta = s e^{-x_2/q}$ (or take what Killing direction will become null). Note that the extension will not work for $q=0$, parallel polarizations.

⊗ How, the blown-up picture looks.



two Null surfaces, in between, to glue up, &
two open regions.
Light cones $\Rightarrow$ white hole, black hole,
like the Schwarzschild Penrose diagram.

⊗ Even better, give a three dimensional picture. $\partial/\partial x^1$ does not appear in the picture, so, we get the entire story.

Tent, rope, weight in the middle. Cross section, hyperbolae hanging from the rope, they generate the interior hyperbolic sheet, exterior flat, asymptotically approach, double tend, region I_o. Exterior region. Take also the opposite tend, like the opposite angle ⇒ the spacetime near $s=0$.

⊗ A two dimensional singularity only in the middle plane, $\xi\eta = -q^2/p^2$, a hyperbola. One-dimensional in the present picture, two dimensional in spacetime, hardly any one will notice it. This is the only curvature singularity.

⊗ Regions I_o, I_e are isometric, ξ, η applies up to the null boundaries, geodesically incomplete. How you extend from there one. Just smoothly, let u, v take negative values as well. Again coordinate singularity ⇒ (ξ, η) extension, keep going, for a maximally analytic extension. 2-dim singularities in all open regions.

⊗ Ted Newman wanted to know how I will get Kerr solution out of colliding waves. Not difficult. <u>Claim:</u> metric in I_o or I_e is locally isometric to Kerr solution. How come, 2 spacelike versus one spacelike – one timelike. It is inside the ergoregion, where Kerr has two spacelike Killing fields. $s=0 \to$ the Cauchy horizon, $u=v=0$ two funny null surfaces inside the ergoregion.

When $q \to 0$, $a \to 0$, $s = 0 \Rightarrow r=0$, real curvature singularity of the Schwarzschild solution, also nothing works for the colliding waves, $R^{\cdots}R_{\cdots} \sim 1/s^{12}$, the strongest rate of divergence that we know so far.

⊗ Another possibility, non smooth extension, the symmetrical story.

Why all that. Not globally hyperbolic, outside of the domain of dependence of initial data prior to the collision not predictability.

⊗ Like any step forward in science, it raises more questions that it answers. What is the typical, the generic singularity structure that would appear in colliding plane gravitational waves. Like previous, compulsory, like now, essentially no singularity? What is generic?. Are they the only two ones, or other kinds also appear?.

⊗ One might think that present solution special, type D. But type D not used very much. Crucial, extension across $u=0$, $v=0$, not the principal null directions. Suggest that one may use the known inventory of stationary-axisymmetric solutions and construct more examples for colliding waves. It does not seem so easy, practically all that I've tried so far fail to admit extensions into region II and III, prior to the collision. For instance, all the T-S except Kerr fail. Except from surface orthogonal Killing fields ⇔ parallel polarizations, all I know,

Nutku-Halil, present, Ehlers transformation to both, no change in the stories, 2 + 2.

⊗ Some physical understanding, distinguishes Nutku-Halil — present solution. Why singularity in colliding plane waves, focusing of the energy, entire ~~the~~ light cone opens versus only one direction. It collects much fewer energy.

⊛ We do not know if our solution is stable. It admits a Cauchy horizon, who knows, it may be that crossing the Cauchy horizon is dangerous, ~~as~~ as it is in Reissner-Nordstrom and Kerr solution.

⊛ Also, I do not know how to treat ~~alm~~ colliding almost plane gravitational waves. At CALTECH, a graduate student of Kip Thorne is apparently studying it.

⊛ Abhay taught me simultaneously ~~two~~ problems. Perfect fluids with less than extreme relativistic equation of state. Actual problem the analytic one, how to solve the equations. The point to curvature singularities on null surfaces.

⊛ Might have some implications, indirect, to cosmic Censorship, by demonstrating ~~can~~ singularities which may arise; they should be taken care of in any formulation.

⊛ Future problems for me?

- One-two more interesting examples with plane gravitationals.
- Couple with electro~~ga~~ magnetic waves. Preliminary results suggest no actual focusing even for parallel polarizations. Also expect, different singularity structures to appear.
- Couple gravitational-hydrodynamic waves. Perfect fluids in region I, extreme relativistic easy, non extreme relativistic, difficult the equations, this is my second problem.

And of course, since this is the Quantum Gravity Seminar I still have to quantize the present solution.

Colliding Gravitational Waves. Santa Barbara, 100486

- Quantum Gravity Seminar
- General problem – Non linear – interesting scattering – Positive – focusing – How much?
- Conjecture, Penrose, Szekeres, curvature singularity inevitable for any observer.
- The message today, it ~~did~~ does not have to be so. Present exact solution. Only very dedicated observers will hit singularity.
- Collaboration with S. Chandrasekhar, U.C.
- Plane waves, 2 Killing fields, hope. Ordinary, sandwich, impulsive.
- Simplest: Impulsive, ~~[illegible]~~ equal magnitude, head on, spacetime.
- Other observers, ~~no~~ different waves, non head on.
- Strategy Penrose, Szekeres, initial data. Characteristic on $u = v = 0$.
- 2 years ago, Chandra, new strategy. Electromagnetic, hydrodynamic, How extend.
- Null separating boundaries, $u = v = 0$, I is $u \geq 0, v \geq 0$, $uH(u)$, $vH(v)$.
- Advantages: C^0 metric ⇒ impulsive waves.

  Demand extendability. – Vacuum equations in regions II, III.
  (only vacuum or ~~electrically~~ flat).
  - Einstein equations on the boundary as distribution.
  - Continuous radiating part of curvature, conservation of energy.
- Not communicate with ~~[illegible]~~ Gods, get results through equation, a few words.
- 2 spacelike Killing fields, $ds^2 = e^{2\Omega}\left[\frac{(d\eta)^2}{1-\eta^2} - \frac{(d\mu)^2}{1-\mu^2}\right] - \sqrt{1-\eta^2}\sqrt{1-\mu^2}\left[\chi (dx^2)^2 + \frac{1}{\chi}(dx^1 - q_2\, dx^2)^2\right]$

  $\chi + i q_2 = Z = \frac{1+E}{1-E}$ ⇒ Ernst equation. Ω quadratures.
- $\Psi = \sqrt{1-\eta^2}\sqrt{1-\mu^2}/\chi$, $\Phi_{,\eta} = \frac{1-\mu^2}{\chi^2} q_{2,\mu}$; $\Phi_{,\mu} = \frac{\sqrt{1-\eta^2}}{\chi^2} q_{2,\eta}$ norm and twist.
- For stationary axisymmetric only norm – twist Ernst equation.
- E = real, hypersurface orthogonal, parallel polarization – static.
- Interesting solution: Khan–Penrose, Nutku–Halil, Kerr, Present, formalism of Chandra + Ferrari.
- Conclusions of analysis of ~~present~~ previous solution solution:

  - Curvature singularity.
  - $u = 1$, $v = 1$, less than four dimensional world, only u = const, null, $\rho_1 = \rho_2 = 0$ timelike,
    physical interpretation.

- Curvature singularity $R^{\cdots\cdot}R_{\cdots\cdot} \sim 1/s^7$, $1/s^3$, less efficient focusing mechanism.

* Feeling – Believe, the singularity is there. 2 years ago, we never contemplated it. More involve situations, Gravitational – Electromagnetic, extremely relativistic fluids, transformation of null dust.
  - Curvature there, non parallel polarizations ⇒ less singular.
- New solution (Tuesday morning) $\mathcal{E} = p\eta + iq\mu$ for norm and twist.
  - No curvature singularity at $u^2+v^2=1$. All Weyl scalars finite. Stuck for 2 months. Chandra 2 days England. EXTENDABLE.
  - Describe what happening: $s = 1-u^2-v^2$, $r = u^2-v^2$, x^2, x^1, ∂/x^2 null. $\xi = s e^{x_2/q}$, $\zeta = s e^{-x_2/q}$, $q=0$ fails.
  - The blown up picture for $s=0$. ~~light~~ null surfaces, light cones, white hole, black hole.
  - Three dimensional picture. The tent. The curvature singularity.
  - I_0, I_e isometric, glued together. ⇒ smooth extensions, maximal analytic extension.
  - Ted Newman: How you get Kerr metric. I_0 locally isometric. Not sure, not surprised. How come, 2 spacelike, 1+1 K.F. Inside the ergoregion.
  - $s=0$ Cauchy horizon, $q=0 \Rightarrow r=0$, real singularity, the strongest.
  - The symmetrical possibility, non-smooth extensions.
  - Outside domain of dependence, A Cauchy horizon, stable or not
  - Raises question? Other forms, which one is generic? (compulsory or not?)
  - Present, type D, not generic. Type not used essentially, $u=v=0$ not principal null directions. Use stationary axisymmetric inventory. Only 2+2, with Ehlers.
  - why, physically works, Opening of light cones at $s=0$.
  - stability for almost plane waves
    - Possible applications to cosmic censorship.

Future, for me: More vacuum examples, with electromagnetism, perfect fluids, less than extreme relativist. And of course, since it is a Quantum Gravity seminar I still have to Quantize them

Colliding waves –

CALTECH 120586

- Broad problem – conjecture – message, much less. – Chandra
- Plane waves, sandwich, impulsive, $u_{ab} + H\ell_a\ell_b$
- Four regions
- old strategy – new strategy – How you extend.
  Null separating boundaries. $uH(u)$, good.
- Gives impulsive – Demand extendability – demand vacuum ⇒ plane impulsive
  No matter everywhere, δ only in Weyl
  H in Ricci but vacuum, nowhere.
  Weyl δ + H, Ricci continuous, Radiating field continuous.

(*) • So far: Kahn – Penrose, surface orthogonal
  Nutku – Halil different polarizations, Szekers, sandwich,

- conclusion  $s = 1 - u^2 - v^2$, $R^{\cdots\cdot} R_{\cdots\cdot} \sim 1/s^7$, $1/s^3$.
  Less efficient focusing.
- Two years ago, never contested the singularity.
- New solution: $u^2 + v^2 = 1$, only coordinate. Find $R_{\cdots}$ finite, next
  extension. $s = 1 - u^2 - v^2$, $\partial/\partial x^2$ null, $\zeta = s\, e^{x_2/q}$, $\zeta = s\, e^{-x_2/q}$
- Blown-up picture ⇒ The tend, The opposite, $\zeta\zeta = -q^2/p^2$
- Further extension, outside domain of dependence, not globally hyperbolic.

(*) The Kerr solution. Chandra + Ferrari wrong impression.
  Equations.
  Locally isometric. How come, Cauchy – horizon., ring singularity.
  $q = 0$, $R_{\cdots\cdot} R^{\cdots\cdot} = 1/s^{12}$

- Question: What is generic, inevitable or not?
- type D, not used much, → i) Not principal
  → ii) Ehlers, still $R_{\cdots}$ finite
  No solution, want surface non-orthogonal.
- Physical distinction Nutku – Halil, present, opening of light cones.
- stability? Within 2 Killing fields, Cauchy horizons
  outside, Ulvi's wrong tells me indications of singularities.
- Other forms of singularities developing?
  $\epsilon = p$ fluids → more
  E – M less singular
  $\epsilon = q + k$, on null surfaces.
- Cosmic censorship?