

# COLLIDING PLANE GRAVITATIONAL WAVES: DO THEY FOCUS OR NOT?

Από τα βιβλία του καθ. Δ. Κωτσάκη ηρωο-  
διδάχτημα αστρονομία, φοιτητής στο Πανεπιστή-  
μιο Θεσσαλονίκης. Η προσωπική μας γνωριμία έγινε  
πολύ αργότερα, λίγα μόλις χρόνια πριν το θάνατό του.  
Μου έκανε ιδιαίτερη εντύπωση ο ζήλος του για την  
αστρονομία, που διατηρήθηκε άεστος μέχρι το  
τέλος της ζωής του, και φαίνεται από το  
αδιάλειπτο συγγραμικό του έργο.

Η είδηση του θανάτου του καθ. Κωτσάκη με  
βρήκε στο Chicago, να ~~αποχαιρέτη με τα φίλα~~  
μελετώ τις προβλέψεις της θεωρίας της σχετιμώ-  
τητας για τις συμφορές βαρυτιμών, υδευτροβαμ-  
τιμών, και υδροδιαριμών (ή αιονισμών) πυρήνων.  
Στη μνήμη του καθ. Κωτσάκη αφιερώνεται το παρόν  
άρθρο επισκόπησης που επισφίζει τα αποτελέσματα  
μιας σειράς εργασιών των τελευταίων τριών  
χρόνων σε συνεργασία με τον S. Chandrasekhar.

## INTRODUCTION

Consider the collision of two gravitational waves in general relativity. Because the theory is non-linear, the waves would not merely pass through each other. Instead one would expect that the waves would scatter each other and interesting phenomena may occur. In fact, because gravity is always attractive, one would expect ~~to~~ ~~see~~ some focusing of the scatter waves to occur. The broad question we are interested is, roughly speaking, how much focusing does general relativity predict?

Within the framework of general relativity one, more generally, could consider the propagation and the collision of some other type of waves like electrodynamic or hydrodynamic (or acoustic) waves. Since these other kinds of waves always result in the creation of gravitational waves as well, the only correct, fully relativistic study of such problems is to consider the collisions of two mixtures of waves, each consisting of gravitational and electromagnetic waves or of gravitational and hydrodynamic waves. A physically interesting question, for instance, is whether the admixture of the electromagnetic or the hydrodynamic waves would make the focusing of the gravitational waves more efficient or not.

Since the early seventies, when such questions were asked for the first time (in the context of the collision of impulsive gravitational waves) through the works of Kahn and Penrose [1] and of Szekeres [2], the "folklore conjecture" is that these collisions always result in the development of curvature singularities and that because these singularities are spacelike, it is inevitable for all observers to fall into these singularities (and get killed) within a finite time from the instance of the collision.

We are still very far from a detailed theory which could predict what would happen when gravitational waves collide. The only theorem, due to Tipler [3], assumes a  $C^2$  spacetime, and as that it is not applicable for the problems we shall be describing. For this reason, we appeal to exact solutions of the Einstein equations which describe such collisions and we draw conclusions from the predictions of these solutions.

The need for exact ~~solutions~~, as opposed to approximate, solutions should be obvious from the type of questions we are considering. Approximate solutions are sufficient when one is pretty much certain about the qualitative

features of the problem and he (or she) actually wants to draw ~~additional~~ quantitative ~~conclusions~~ ~~statements~~ results. But it is not accustomed in studies of general relativity that intuition might be misleading while the theory is richer than our imagination. For the present state of understanding of general relativity, therefore, exact solutions seem to be the only safe recipe for the study of the collision of gravitational waves.

Because the Einstein equations are quite involved, it is not generally easy to formulate interesting physical problems which can be treated exactly in general relativity. However, such a problem does exist for the collision of waves: the collision of plane gravitational waves, possibly coupled with electromagnetic or hydrodynamic waves. Consider for the time being, that the <sup>plane</sup> waves collide "head on". Since the geometry of the spacetime is invariant in the two-dimensional planes parallel to the two wave fronts, the interaction region of the spacetime, to the future of the collision should also possess this same symmetry. On physical grounds, therefore, we expect that the interaction region (and as a consequence, the entire spacetime) should possess two space-like

commuting killing fields and we impose these symmetries on the spacetime metric. Fortunately, a great deal of progress has been made in the late seventies [4] about constructing solutions of the Einstein equations with two commuting killing fields.

the interaction region, which carries two spacetime commuting Killing fields, ~~great~~ <sup>has been</sup> ~~made~~ <sup>in the</sup> ~~late~~ <sup>late</sup> ~~seventies~~ <sup>seventies</sup> in constructed solution of the Einstein equation with the commuting Killing fields.

- What kind of waves.

Gen. Rel. predicts the existence of plane and sandwich and impulsive waves propagating in flat space. The <sup>deep</sup> reason for existence of sandwich waves is the fact that plane waves are of the Kerr-Schild form,  $g_{ab} = \eta_{ab} + f \ell_a \ell_b$  where  $\eta_{ab}$  is flat and  $\ell_a$  null (with respect to both  $\eta_{ab}$  and  $g_{ab}$ ) and the <sup>generally non-linear</sup> ~~Einstein~~ ~~vacuum~~ equations become linear for  $f$ . The sandwich waves are described by a slab of curvature propagating in flat spacetime (Fig. 1). when the width of the sandwich wave tends to zero and its amplitude to a ~~delta~~ Dirac  $\delta$ -function, we get an impulsive <sup>plane</sup> wave ~~propagating~~ propagating in flat space.

The spacetime describing the collision of two plane gravitational waves consists of four regions (Fig. 2). Region IV is flat, and represents the ~~part of the~~ portion of the spacetime before the arrival of neither wave. The propagation of the waves