

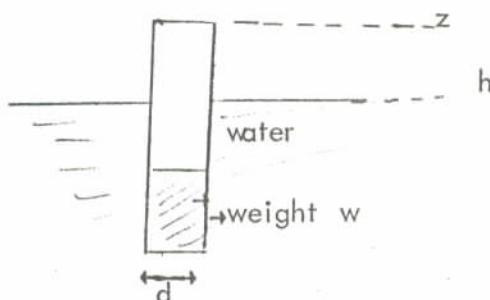
JUNE, 1975

1.

A rocket starts out at rest with total mass  $M_0$ . This includes the fuel. The exhaust velocity (relative to the rocket) has the constant value of  $u$ . When the rocket engine cuts off, the final mass is  $M$ . What fraction of the energy generated by the rocket motor has gone into the kinetic energy of the rocket? Neglect gravity. (NOTE: Present day rockets do not travel with relativistic velocities.)

2.

Determine the vertical motion of a cylindrical bobber (see figure) which is steadily riding a surface wave, the latter being represented by a periodic change of the surface level, given by the formula,  $h = a \cos(\omega t)$ . Assume that the frictional forces are small but nonzero.



3.

Particle accelerators of the ring type keep the accelerating particles in orbit by sets of magnets. (i) What is the required peak magnetic field strength for an accelerator 1 Km in radius which accelerates protons to 300 Gev? How about a similar machine for electrons? (ii) Charged particles moving in the circular orbit lose energy by radiation. Compute the energy loss per revolution due to radiation and again compare the result for protons and for electrons.

Hints:

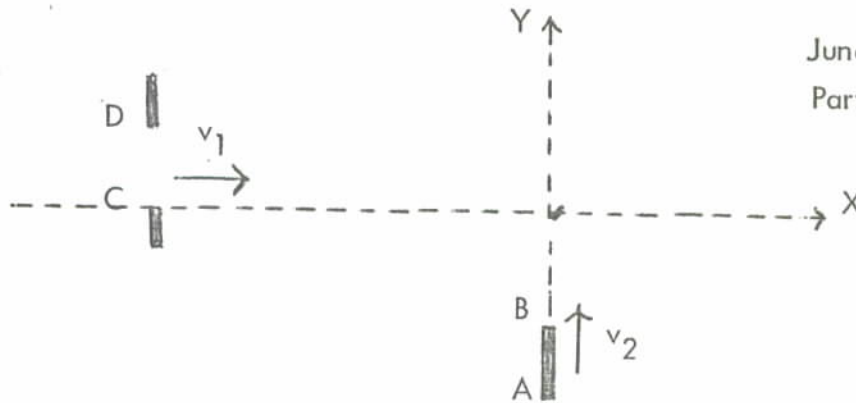
(i)  $1 \text{ Gev} = 10^9 \text{ ev} = 1/3 \times 10^7 (e \times \text{stat volt})$  where  $e$  is the electron charge.

(ii) Rate of radiation of charged particle in motion:

$$\frac{d\epsilon}{d\tau} = \frac{2}{3} \frac{e^2}{c} \left( \frac{d u^\mu}{d\tau} \frac{d u_\mu}{d\tau} \right)$$

where  $u^\mu = \frac{dx^\mu}{d\tau}$  is the 4-velocity and  $\tau$  the proper time of the particle.

4.



Consider the system depicted in the above figure. A stick AB (of length  $\ell$ ) moves along the  $y$ -axis with constant velocity  $v_2$ . A disc with a hole of diameter CD (equal to  $\ell$ ) moves along the  $x$  axis with constant velocity  $v_1$ . It is so arranged that points A and C will pass the origin at the same time.

Write down and/or draw the trajectories of A, B, C, D in at least two of the three relevant inertial frames (i) the Lab. frame, (ii) the rest frame of AB and (iii) the rest frame of CD. (Relativistic effects are, of course, assumed to be important)

Which of the following <sup>20 points</sup> naive arguments is false? (i) To an observer at rest with AB, the moving hole CD is Lorentz contracted; it cannot get past AB. (ii) To an observer at rest with CD, the moving stick AB is contracted and will pass the hole with no difficulty.

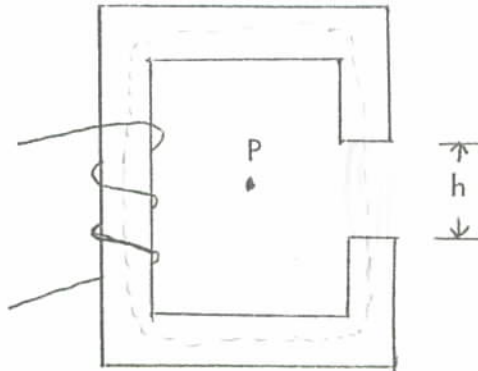
5.

An atom of rest mass  $M$  when it is in the ground state is in a uniform gravitational field of strength  $g$ . Initially it is at a height  $\ell$  and in an excited state of excitation energy  $E$ . It is then allowed to drop to the ground level, whereupon: (i) it de-excites to the ground state, (ii) it is bounced back with a velocity such that it will just get back to the original height, and (iii) a photon with the balance of the energy is emitted in the vertical direction. (The earth takes up the momentum transfer with negligible energy pick up.)

- What is the frequency of the photon when captured by a detector near the ground level?
- What is the frequency of the photon if it is captured by a detector at height  $\ell$ ?
- Demonstrate why it would be inconsistent if one should assume that the frequency of the photon is unaffected by the gravitational field.

6.

A long iron rod of cross section  $A$ , length  $L$  and magnetic permeability  $\mu$  is bent as shown, with a small air gap (distance =  $h$ ). A coil is wound on the rod  $N$  times and energized with a time dependent current  $I(t) = b t$ . What is the total work required to move a point charge  $q$  along a straight line (from  $-\infty$  to  $+\infty$ ) normal to the paper at point  $P$ .



7.

Two parallel beams of charges, each having a charge density  $\lambda = 10^{-6}$  Coulomb/cm travel with velocity  $v_x$  in the  $x$ -direction. The separation between the beams is  $D = 2$  cm. Calculate the direction and magnitude of the force per unit length experienced by each beam when

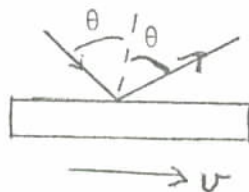
a)  $v_x = \frac{1}{2} c$

b)  $v_x = c$

( $c$  is the velocity of light)

8.

A mirror is moving with velocity  $v$  in a direction parallel to the mirror face. Show that for reflection of light off this mirror the angle of incidence is equal to the angle of reflection, correct to all orders in  $v/c$ .



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PH.D. CANDIDACY EXAMINATION

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9.

Explain the observation of the rainbow. Does its size depend on the time of day? Explain.

10.

When a rubber band is stretched adiabatically it becomes warmer. Using this fact, show that if a rubber band is held stretched at a fixed length and heated, the tension must increase.

11.

There is a theory which asserts that the quarks obey parafermi statistics of order 3. In simple terms, it means that each quantum state of the quark may be occupied up to three times. Following the standard techniques used in statistical mechanics, derive the distribution function  $f$  for a quark gas.

[The number  $dN$  of quarks of each kind and spin direction within a phase space volume element  $d\Omega$  is given by  $dN = f d\Omega$  ]

12.

Solve:

$$f(z) = z^2 + f(z^2)$$

$$f(0) = 1$$

and show that the singularities of  $f(z)$  lie dense everywhere on the unit circle  $|z| = 1$  in the complex plane.

13.

Calculate the energy gap for electrons in a weak one-dimensional periodic potential

$$V = V_0 \cos(2\pi x/a)$$

14.

Calculate (i) the Fermi energy  $E_F$  (in eV), (ii) the Fermi temperature  $T_F$  and (iii) the magnetic moment  $M$  per unit volume in a magnetic field  $B = 10^4$  gauss of a system of  $n = 10^{22}$  electrons per  $\text{cm}^3$  in a box near  $T = 0$ .

15.

When measuring a scattering cross-section with a liquid hydrogen target, one must take some data with an empty target in order to measure the background. If the signal to background is expected to be approximately 10 to 1, what fraction of the running time should be spent with empty target in order to minimize the uncertainty in the calculated cross-section?

16.

Describe the principal components of a frequency to voltage converter circuit, whose output voltage is proportional to the frequency of the sinusoidal input and independent (within a reasonable range) of the magnitude of the input voltage.



17.

Consider a particle of mass  $m$  in an infinitely deep square well in one dimension. The width of the well is  $a$ . Repeated measurements of the kinetic energy on identically prepared systems yield a value

$$E = \pi^2 \hbar^2 / 2 m a^2$$

in one half of the trials and

$$E = 4 \pi^2 \hbar^2 / 2 m a^2$$

in the other half. [ Here energy is measured with respect to the bottom of the well] Further, at time  $t=0$  the expectation value of the momentum is  $8\hbar / 3a$ .

What is the wave function  $\psi(x, t)$  of the system?

18.

A non-relativistic spin  $\frac{1}{2}$  particle is in a potential

$$V = V_1(r) + \vec{S} \cdot \vec{L} V_0,$$

where  $\vec{S}$  and  $\vec{L}$  are the spin and orbital angular momentum operators respectively.

The wave function of an arbitrary state  $|\psi\rangle$  is  $\psi(\vec{r}, s_z) = \langle \vec{r}, s_z | \psi \rangle$

(i) Write down a maximal set of commuting operators which include the Hamiltonian.

(ii) Label the stationary basis vectors corresponding to the eigenvalues of these operators and specify the eigenvalues except for the Hamiltonian.

(iii) Write down (not derive) the coordinate space angular wave function for these states in terms of spherical harmonics and Pauli spinors. Derive the Schrödinger equation for the corresponding radial wave function.

(HINT: Write  $p^2 = p_r^2 + \frac{\hbar^2}{r^2} L^2$  and  $p_r = \frac{\hbar}{i} \left( \frac{1}{r} + \frac{\partial}{\partial r} \right)$  in the coordinate representation.)

19.

The low energy neutron-proton scattering amplitude is given by the formula

$$A = \alpha + \beta \vec{\sigma}_n \cdot \vec{\sigma}_p$$

where  $\vec{\sigma}_n$  and  $\vec{\sigma}_p$  are the Pauli spin operators for neutron and proton respectively.

Derive the cross section for the scattering of an unpolarized beam of neutrons with an energy equivalent to  $1^\circ \text{K}$  by a hydrogen molecule ( $\text{H}_2$ ) under each of the following conditions

	Initial state of $\text{H}_2$	Final state of $\text{H}_2$
a)	$S = 1$	$S = 1$
b)	$S = 0$	$S = 0$
c)	$S = 1$	$S = 0$
d)	$S = 0$	$S = 1$

(  $S$  designates the total nuclear spin of the hydrogen.)

20.

The spectral line of mercury at  $1850 \text{ \AA}$  splits under the influence of a magnetic field of  $10^3$  gauss into three components separated by  $0.0016 \text{ \AA}$  intervals. Determine whether this Zeeman effect is normal or anomalous and thereby determine the orbital or spin character of the current affected by the field.

21.

Calculate and sketch the temperature dependence of the spin contribution to the heat capacity of a paramagnetic material ( $S = \frac{1}{2}$ ) containing  $10^{21}$  spins per  $\text{cm}^3$  in a magnetic field of  $B = 10^4$  gauss. The coordinates of your sketch should have scales accurate to within a factor 2.

$g = 2 \leftarrow \text{given}$

22.

Consider the possibility of producing a new particle X having a mass of  $10 \text{ GeV}/c^2$  by protons on a lead (Pb) target. Estimate the threshold (minimum) proton energy required under each of the following assumptions.

a)  $p + \text{Pb} \rightarrow p + \text{Pb} + X$

b)  $p + N \rightarrow p + N + X$  where N is a nucleon inside a Pb nucleus. Regard the nucleus as a Fermi gas of quasi-free nucleons.

$A_{\text{Pb}} \approx 208$

23.

We want to determine whether the newly discovered  $\psi$  particle decays via the strong interaction. To begin, we make a list of the decay modes which are allowed or forbidden according to the strong interaction selection rules. Assuming the  $\psi$  has a mass of  $3100 \text{ MeV}/c^2$ ; a spin parity  $1^-$ ; isotopic spin = 0; baryon number = 0; indicate which of the following are forbidden; and why

- ✓ a)  $\psi \rightarrow p\pi^-$  violates  $N_B$  conservation.
- ✓ b)  $\psi \rightarrow \bar{p}p\bar{p}p$   $M_\psi = 938 \times 2 \times 2 \times 2 \text{ MeV}$  violates energy conservation + momentum.
- c)  $\psi \rightarrow \pi^- \pi^+ \pi^0$
- ✓ d)  $\psi \rightarrow \pi^+ \pi^-$   $I=1, l=1$ , violates isospin conservation
- e)  $\psi \rightarrow \bar{p}p$   $I=0, S=0, l=1$  violates parity, can go through  $S=1, l=0$
- ✓ f)  $\psi \rightarrow \pi^0 \pi^0 \pi^0$   $\pi^0, \pi^0: I=1, l=1 \Rightarrow$  use fm of  $\pi^0\pi^0$  anti-symmetry violates Pauli principle

$(-1)^l = +1, S=0.$

$\pi^- \pi^+ :$   
 $l=1$   
parity  $-1$   
 $I=1$

$\pi^0 \pi^0$   $I=1$  only.  
 $l=1$



June, 1975

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24. Choose the correct answers for each of the following questions. (There may be more than one in some cases.)

1) When a fast charged particle goes through matter it suffers energy loss, this is mainly due to collisions with : (a) electrons ; (b) nuclei.

The change in the trajectory of the charged particle is due mainly to collisions with : (c) electrons ; (d) nuclei.

2) A  $\mu^-$  meson is captured by a lead nucleus in a 1 S orbit. Let  $R$  = radius of the nucleus,  $a_\mu$  = radius of the  $\mu^-$  orbit,  $a_e$  = radius of orbit of an 1S electron and  $\lambda_e$  = Compton wavelength of the electron, then :

(a)  $R < \lambda_e < a_\mu < a_e$  ; (b)  $a_\mu < R < \lambda_e < a_e$  ;  
(c)  $R < a_\mu < \lambda_e < a_e$  ; or (d)  $R < \lambda_e < a_e < a_\mu$ .

3) The wavelength of a thermal neutron is nearest to: (a)  $10^{-2}$  cm ; (b)  $10^{-5}$  cm ; (c)  $10^{-8}$  cm ; or (d)  $10^{-11}$  cm.

4) The specific heat (in units of cal/gram degree) of common rock is nearest to: (a)  $10^{-1}$  ; (b) 1 ; (c) 10 ; (d) 100.

5) Which of the following can in principle cause the orbit of a planet around the sun to precess? (a) Finite mass of the sun ; (b) Finite size of the sun ; (c) Non-spherical shape of the sun ; (d) Relativistic effects.

6) Isospin conservation implies : (a)  $\sigma_{pp} = \sigma_{nn} = \sigma_{pn}$  ; (b)  $\sigma_{pp} = \sigma_{nn}$  ; (c)  $\sigma_{pn} = 2\sigma_{pp}$  .  
( The  $\sigma$ 's are scattering cross sections.)

7) The boiling temperature of nitrogen is about :

(a)  $10^\circ\text{K}$  (b)  $40^\circ\text{K}$  (c)  $80^\circ\text{K}$  (d)  $120^\circ\text{K}$  (e)  $160^\circ\text{K}$  .

- 8) The interaction mean free path of a 1 Gev neutron in the air is:  
 a)  $\sim 5$  cm; (b)  $\sim 5$  m; (c)  $\sim 500$  m.

- 9) An electron enters a long Čerenkov counter.



Does the electron or Čerenkov light reach the far end of the counter first?

- (a) The electron; (b) The Čerenkov light; (c) The electron and Čerenkov light reach the far end simultaneously.
- 10) A 4000 lb Cadillac automobile is parked on 57th Street. What surface area on each tire is touching the ground? (Assume that the tires are inflated to a pressure of 30 lbs/in<sup>2</sup>.) (a) 25 in<sup>2</sup>; (b) 35 in<sup>2</sup>; (c) 45 in<sup>2</sup>.

particle energy of the order of the mass  $mc^2$   
 slarger  $\Rightarrow$  relativistic.

The following list of constants may be used for computing problems:

Acceleration due to gravity at surface of earth	g	980 cm/sec <sup>2</sup> or 32.2 ft/sec <sup>2</sup>
Gravitational constant		$6.67 \times 10^{-8} \frac{\text{dyne cm}^2}{\text{gm}^2}$
Radius of the earth		4000 mi or 6370 km
Volume of one mole of gas at STP		22.4 liters
Boltzmann constant	k	$1.37 \times 10^{-16} \frac{\text{ergs}}{\text{atom } ^\circ\text{K}}$
Gas constant per mole	R	$8.31 \times 10^7 \frac{\text{ergs}}{\text{mole } ^\circ\text{K}} = 2 \text{ cal/mole } ^\circ\text{K}$
Avogadro's number	No	$6.02 \times 10^{23}$
Velocity of light	c	$3 \times 10^{10} \text{ cm/sec}$
Pressure of 1 standard atmosphere		$15 \text{ lb/in}^2 = 10^6 \text{ dynes/cm}^2 = 760 \text{ mm of Hg}$
Density of air STP		1.29 gm/liter
Charge of electron	e	$1.6 \times 10^{-19} \text{ coulombs}$
Charge of electron	e	$4.8 \times 10^{-10} \text{ e.s.u.}$
Specific electronic charge	e/m	$1.76 \times 10^8 \text{ coulombs/gm}$
1 ev		$1.602 \times 10^{-19} \text{ joule}$
Ratio mass of proton to mass electron		1840
Mass of proton		1.00760 amu = 938.24 Mev.
Mass of electron	0.00055 amu	m $9.1 \times 10^{-28} \text{ gm} = 0.511 \text{ Mev}$
Mass of neutron		1.00898 amu = 939.52 Mev
1 atomic mass unit		$1.66 \times 10^{-24} \text{ gm} = 931.16 \text{ Mev}$
Planck's constant of action	h	$6.6 \times 10^{-27} \text{ erg sec}$
Bohr radius (n = 1 for H) $\hbar^2/me^2$		0.53 Å
One Bohr magneton $\mu = e\hbar/2mc$		$0.927 \times 10^{-20} \text{ ergs/gauss}$
Radiation density constant	a	$7.6 \times 10^{-15} \text{ erg cm}^{-3} \text{ deg}^{-4}$
Wave number $(1 \text{ cm})^{-1}$ corresponds to		$1.99 \times 10^{-16} \text{ ergs}$
Stefan-Boltzmann constant		$5.7 \times 10^{-5} \text{ erg cm}^{-2} \text{ deg}^{-4} \text{ sec}^{-1}$
Rydberg constant	$R_\infty$	$109,737 \text{ cm}^{-1}$
Energy conversion	J	4.19 joules/cal
Magnetic moment of proton		2.79 nucleon magnetons

(3).  $E = \frac{m}{\sqrt{1-v^2}}$ ,  $\frac{dE}{dt} = \vec{F} \cdot \vec{v} = \frac{q}{c} \vec{v} \cdot (\vec{v} \times \vec{B}) = 0$

$\vec{F} = \frac{q}{c} (\vec{v} \times \vec{B})$  the energy, ~~and~~ without radiation, remains constant

$$E' = \gamma (\vec{E}_{\perp} + \frac{\vec{v}}{c} \times \vec{B}_{\perp})$$

$\vec{p} = \frac{m\vec{v}}{\sqrt{1-v^2}}$   $E = \frac{m}{\sqrt{1-v^2}} = \text{const} \Rightarrow v^2 = \text{constant}$

Eq. of motion.

$$\frac{d\vec{p}}{dt} = \vec{F} \Rightarrow \frac{d}{dt} \left( \frac{m\vec{v}}{\sqrt{1-v^2}} \right) = \frac{q}{c} \vec{v} \times \vec{B} \Rightarrow$$

$$\frac{m}{\sqrt{1-v^2}} \frac{d\vec{v}}{dt} = \frac{q}{c} \vec{v} \times \vec{B}, \text{ in two dimensions } \therefore$$

$$\begin{cases} \frac{m}{\sqrt{1-v^2}} \frac{dv_x}{dt} = eB v_y \\ \frac{m}{\sqrt{1-v^2}} \frac{dv_y}{dt} = -eB v_x \end{cases}$$

we can solve them using  $v_x + i v_y$ .

or we know that the motion will be  $v_x = \alpha \sin \omega t$ ,  $v_y = \alpha \cos \omega t$ , substitution gives  $\omega = \frac{eB}{m} \sqrt{1-v^2}$



$$w = \frac{eB}{m} \sqrt{1-v^2} \quad v = wR \Rightarrow R = \frac{mv}{eB \sqrt{1-v^2}} \quad \text{or}$$

$$\boxed{R = \frac{pc}{eB}}, \text{ the same formula, if we work it classically.}$$

From  $E^2 = p^2 c^2 + m^2 c^4$ ,  ~~$m^2 c^4$~~  forget in both cases,  $E = pc = 300 \text{ GeV} \Rightarrow$

$$B = 10^4 \text{ Gauss.}$$

$$\frac{d\varepsilon}{dt} = - \frac{d\varepsilon_{\text{rad}}}{dt} = - \frac{2}{3} \frac{e^2}{c^3} \left( \frac{d\vec{v}}{dt} \right)^2$$

$$v^\mu = \frac{dx^\mu}{dz} = \frac{d}{dz} (\vec{x}, i ct) = \left( \frac{d\vec{x}}{dz}, i \gamma \right) = \gamma \left( \frac{d\vec{x}}{dt}, i c \right)$$

$$\frac{dv^\mu}{dt} = \gamma^2 \left( \frac{d^2 \vec{x}}{dt^2}, 0 \right) \Rightarrow \frac{d\varepsilon}{dt} = \frac{2}{3} \frac{e^2}{c^3} \gamma^4 \left( \frac{dv}{dt} \right)^2$$

$$\frac{dv}{dt} = \frac{1}{\gamma m} \left( \gamma m \frac{dv}{dt} \right) = \frac{1}{\gamma m} \left( F_{\text{force}} \right)^2$$

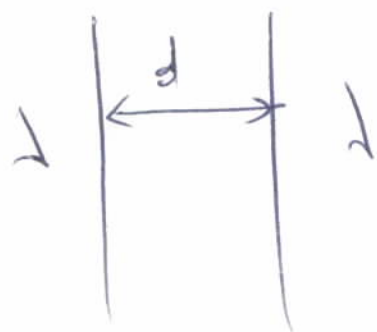
and  $\boxed{\text{since } \gamma \text{ is independent of } t.}$   

$$F^2 = q^2 (\vec{v} \times \vec{B})^2$$



⑦. \_\_\_\_\_  
 \_\_\_\_\_

$\lambda$  density,  $v$  velocity  
 in the system with  
 velocity  $v$ :



$F = \frac{2\lambda^2}{d}$  in the system of rest.



electric + magnetic field.

$F_E = \frac{2\lambda'^2}{d}$ ,  $\lambda'$  different,

$\lambda = \frac{dq}{dx}$

$\lambda' = \frac{dq}{dx'}$

$\lambda' > \lambda$

$\lambda' = \lambda \gamma$

Magnetic field.

$\lambda = \frac{dq}{dx}$

$I$

$I = \lambda v$

ampère,

$B \cdot 2\pi r = \frac{\mu_0}{c} I \Rightarrow$

$B = \frac{2I}{cd}$

$F = \frac{dq}{c} (\vec{v} \times \vec{B}) = \frac{\lambda' dx'}{c} (\vec{v} \times \vec{B})$

$F = \frac{2\lambda'^2}{2d} = \frac{2\lambda'^2 v^2}{2d}$

$$F_{\text{tot}} = F_E - F_B = \frac{2\lambda'^2}{d} \left(1 - \frac{v^2}{c^2}\right)$$

substitute  $\lambda' \Rightarrow$

$F_{\text{tot}}$  independent of  $v$

$$F_{\text{tot}} = \frac{2\lambda^2}{d}$$

⑩.  $P \rightarrow -F$   
 $V \rightarrow L.$

hypothesis  $\left(\frac{\partial T}{\partial L}\right)_S > 0.$

Maxwell  $\left(\frac{\partial T}{\partial V}\right)_S = - \left(\frac{\partial P}{\partial S}\right)_V \Rightarrow$

$\left(\frac{\partial F}{\partial S}\right)_L > 0, \quad L \text{ the tension.}$

(11) 
$$Z_{\text{fermi}} = 1 + e^{\beta(\mu - \epsilon_r)}$$

\_\_\_\_\_  $Z$   $\epsilon_r$  is energy plus offset.  
 $n_r$  = occupation number.

If we have a lot of states,

$$N = \sum_r n_r$$

$$E = \sum_r n_r \epsilon_r$$

$$Z = \text{Tr } e^{-\beta(\epsilon - \mu)} = 1 + e^{\beta(\mu - \epsilon_r)}$$

For Bosons  $Z = 1 + e^x + e^{x^2} + e^{x^3} + \dots$

$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu)} + 1}$$

$$N = \int f(\epsilon) \rho(\epsilon) d\epsilon$$

$\swarrow$  distribution function.       $\searrow$  density of states

$$N_r = - \frac{\partial(\ln Z)}{\partial(\beta \epsilon_r)}$$

$$- \beta \epsilon_r = \ln Z = \ln(1 + e^{\beta(\mu - \epsilon_r)})$$

the number of particles  
in the  $r$ -state

$$N_r = \frac{e^{\beta(\mu - \epsilon_r)}}{1 + e^{\beta(\mu - \epsilon_r)}} \leadsto n_r = \frac{1}{e^{\beta(\epsilon_r - \mu)} + 1}$$

$$N = \sum \frac{1}{e^{\beta(\epsilon_i - \mu)} + 1} \sim \int f(\epsilon) \rho(\epsilon) d\epsilon = \int \frac{1}{e^{\beta(\epsilon - \mu)} + 1} \rho(\epsilon) d\epsilon$$

$$\frac{dN}{d\epsilon} = f(\epsilon) \rho(\epsilon).$$

$$\text{Quanta, } Z = 1 + e^{\beta(\mu - \epsilon_1)} + e^{\beta(\mu - \epsilon_2)} + \dots$$

$$N = \frac{1}{\beta} \ln \left( 1 + e^{\beta(\mu - \epsilon_1)} + e^{\beta(\mu - \epsilon_2)} + \dots \right)$$

$$f \sim - \frac{1}{\beta} \ln \left( 1 + e^{\beta(\mu - \epsilon_1)} + e^{\beta(\mu - \epsilon_2)} + \dots \right)$$



(13).  $V = V_0 \cos\left(\frac{2\pi x}{a}\right)$ .

$H = \frac{p^2}{2m} + V_0 \cos\left(\frac{2\pi x}{a}\right)$  unperturbed  $\psi_0 = \frac{1}{\sqrt{2\pi}} e^{ikx}$  or

$\psi = \frac{1}{\sqrt{V}} e^{ikx}$  normalized in volume  $V$ .

$E = \frac{\hbar^2 k^2}{2m}$  perturbation

$$\langle k' | V | k \rangle = \frac{V_0}{V} \int e^{-ik'x} \cos\left(\frac{2\pi x}{a}\right) e^{ikx} dx =$$

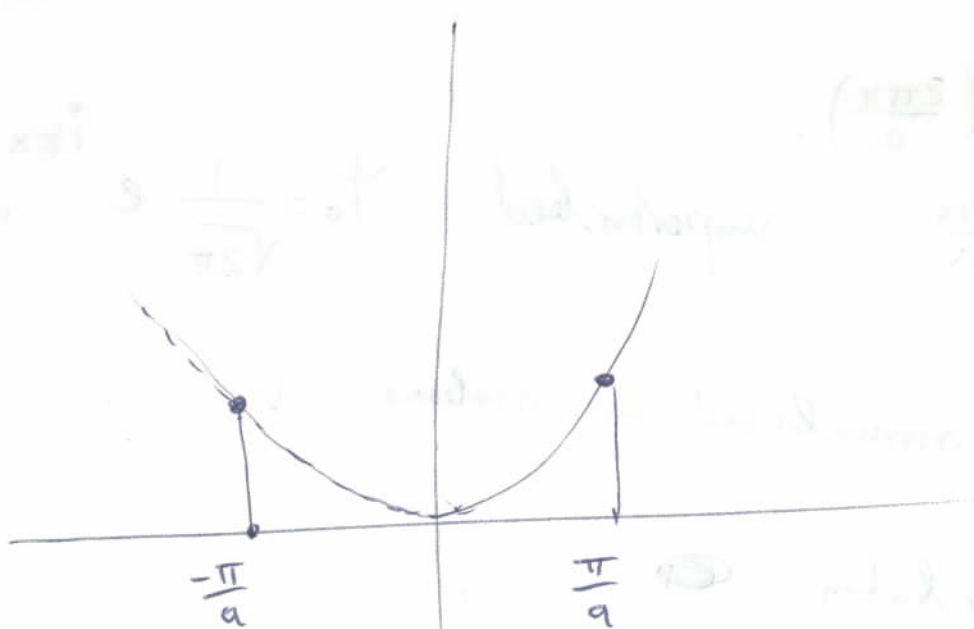
$$= \frac{V_0}{2V} \int_0^V e^{i(k-k')x} \left[ e^{i\frac{2\pi x}{a}} + e^{-i\frac{2\pi x}{a}} \right] dx = \left( \frac{2\pi}{a} = k_0 \right)$$

$$= \frac{V_0}{2V} \left\{ \int_0^V e^{i(k_0+k-k')x} dx + \int_0^V e^{i(k-k'-k_0)x} dx \right\} =$$

~~$\frac{V_0}{2V} \left\{ \int_0^V e^{i(k_0+k-k')x} dx + \int_0^V e^{i(k-k'-k_0)x} dx \right\}$~~   $\frac{V_0}{2} \left\{ \delta(k_0+k-k') + \delta(k-k_0-k') \right\},$

non zero only for  $\begin{cases} k_0+k=k' \\ k-k_0=k' \end{cases} \Rightarrow$

$$\begin{cases} \frac{2\pi}{a} + k = k' \\ \frac{2\pi}{a} + k' = k. \end{cases}$$



$$k, k'$$

non zero if  $|k - k'| = \frac{2\pi}{a}$

degenerate  $k, k'$

$k = \frac{\pi}{a}, k' = -\frac{\pi}{a}$  or opposite.

$$\begin{pmatrix} 0 & \frac{V_0}{2} \\ \frac{V_0}{2} & 0 \end{pmatrix}$$

$$\Delta E = \pm \frac{V_0}{2} \Rightarrow$$

energy gap  $V_0$ .

$$H = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} & \frac{V_0}{2} \\ \frac{V_0}{2} & \frac{\hbar^2 k'^2}{2m} \end{pmatrix}$$

(14)

$$\begin{cases} B = 10^4 \text{ gauss} \\ \eta = 10^{22} \\ T = 0. \end{cases}$$

$$\uparrow \uparrow \uparrow B$$

$$M \approx \frac{N \mu^2 B}{kT} \cdot \frac{T}{T_F} \Rightarrow$$

$$M \approx \frac{N \mu^2 B}{k T_F}.$$

$$\mathcal{E}_+ = -\vec{\mu} \cdot \vec{B}, \quad \vec{\mu} = -\frac{e\hbar}{2mc} \vec{S}, \quad \mathcal{E}_+ = \frac{e\hbar}{2mc} \vec{S} \cdot \vec{B} \Rightarrow$$

$$\mathcal{E}_+ = \frac{e\hbar}{2mc} B \cdot \frac{1}{2}, \quad \mu_B = \frac{e\hbar}{2mc}, \quad \mathcal{E} = \pm \mu_B B$$

particles // spin

$$N_+ = \int_0^\infty f(\mathcal{E} - \mu_B B) \rho(\mathcal{E}) d\mathcal{E}$$

$$N_- = \int_0^\infty f(\mathcal{E} + \mu_B B) \rho(\mathcal{E}) d\mathcal{E}.$$

magnetic  $M = \mu_B (N_+ - N_-) = \mu_B \int f(\mathcal{E} - \mu_B B) - f(\mathcal{E} + \mu_B B) \rho(\mathcal{E}) d\mathcal{E} =$

$$= 2 \mu_B^2 B \int -\left(\frac{\partial f}{\partial \mathcal{E}}\right) \rho(\mathcal{E}) d\mathcal{E}$$

$$T=0, \quad \frac{\partial f}{\partial \mathcal{E}} = \delta(\mathcal{E})$$

(15)

$$I = \frac{N}{t} \Rightarrow N = It \quad I = \text{intensity}$$

If  $\propto I$  : ~~so~~ number of particles which scatter per unit time

$(1-\alpha)I$  = do not interact.

(22)  $p + Pb \rightarrow p + Pb + X:$

$M_X = 10 \text{ GeV}/c^2$

$P_i = (\vec{p}_i, i(E_i + m_i))$

Lab system:

$P_{f, \text{c.m.}} = (0, i(m_1 + m_2 + m_X))$

Conservation:

$$P_1^2 - E_1^2 - m_1^2 - 2E_1 m_2 = - \frac{(m_1 + m_2 + m_X)^2}{219}$$

$\begin{matrix} \nearrow 1 \text{ GeV} & \nearrow 10 & \nearrow 208 \\ & & \end{matrix}$

$P_1^2 - E_1^2 - m_1^2 - 2E_1 m_2 = - 219^2 \Rightarrow$

$+ m_1^2 + m_2^2 + 2E_1 m_2 = + 219^2$

$E_1 = \frac{219^2 - 1^2 - 208^2}{2 \cdot 208} = \frac{(219 + 208) \cdot 11}{2 \cdot 208} \approx 11$



(22b)

$P_1, m_1$

$\xrightarrow{P_1}$   
 $m_1$

$\xleftarrow{N_q}$



$N$  energy fermi

$$|P_f' = (\vec{P}_1, i(E_1 + E_{kin}))$$

$$P_f' = (0, i(m_1 + m_1 + m_x))$$

$$E_F^0 = \frac{\hbar^2}{2m} \left( \frac{3\pi^2 N}{V} \right)^{2/3}$$

$$V = \frac{4\pi}{3} R^3$$

$$R = 1.2 A$$

The Fermi energy of the nucleus  
is of the order of eV.

$$\Psi: \pi^+ \pi^-$$

$$I=0$$

$$\Psi_{\text{isosp.}} = +$$

$$\text{parity} = (-1)^2 (-1)^l \Rightarrow l \text{ odd}$$

$$l=1,$$

$$\Psi \rightarrow p \bar{p} \quad p$$

permitted

$l$  even

unpolarized spins

There is no interference

$$10^{-4}$$

2