COLLOQUIUM

ΦΥΣΙΚΟΥ ΤΜΗΜΑΤΟΣ

Ομιλητής: Καθ. Sabrahmanyan Chandrasekhar
University of Chicago

Θέμα: The two center problem in general relativity

Τόπος: Αίθουσα Σεμιναρίων (Λ-207)

Χρόνος: Πέμπτη 6 Απριλίου, 5:00 μ.μ.

Iraklion, 06 04 89.

Chandra: The two center problem in general relativity.

It is a big honor for the University of Crete and the Research Center of Crete that Prof. Chandrasekhar is giving the colloquium today.

Prof. Chandrasekhar needs no formal introduction and I am not going to provide one. He is one of the most distinguished ~~theoretical~~ astrophysicists of this century and I think I could say that he is the man who made theoretical astrophysics an exact science.

To our students I would only say that it is not that bad even if they could not follow his entire lecture. It should feel good to recall, in later days, that "while a student at the University of Crete I did attend a lecture by Chandra". And that I've leant about the Chandrasekhar limit while a freshment and I did heard Chandra himself while I was a junior or a senior.

Chandra is going to talk today about the two-center problem in general relativity.

Thank you very much Chandra, we are very honored.

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FOR THE ATTENTION OF: Prof. S. Chandrasekhar

AT: LASR

FAX ADDRESS: 001-312-702 6645

SENT BY: Basilis Xanthopoulos

NUMBER OF PAGES TO FOLLOW: —

DATE: June 9, 1989.

MESSAGE

Dear Chandra:

My summer schedule is taking shape. I will be arriving in Chicago on July 9th and I will be leaving on July 22nd.

Thanasis Economou (who tells me you got some nice statues from Greece) will make arrangements for my stay at the International House. Yours, Basilis.

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Laboratory for Astrophysics and Space Research

March 16, 1989

Professor Basilis Xanthopoulos
Department of Physics
University of Crete
714-09 IRAKLION, CRETE (GREECE)

Dear Basilis,

Herewith the proofs of the two papers. I know that you read the proofs very carefully, so there is no reason for me to say anything in that connection. I should be grateful if you could send your corrections by FAX so that I can receive them at least by Monday morning. Neither I nor Mavis has spent any time on the proofs. We shall do so before your corrections arrive.

As for our forthcoming trip to Greece: there have been some slight changes. Mr. Ammerman called to say that the people in Salonika would like me to visit them; and I have agreed to go to Salonika on the morning of Thursday, March 30th, repeat my Athens lecture on the 31st, leave Salonika on the morning of Saturday, April 1, and arrive in Heraklion the same day. I do not know the corresponding flight times but Ammerman said he would arrange for the reservations and the tickets and you can probably find out from him the itinerary.

The other part of the change refers to the date of our leaving Heraklion. The schedule for that is enclosed. I should be grateful if you will transmit the computer print-out to Ammerman who has promised us to obtain the tickets for the flights that have been reserved already in my and Lalitha's names.

As you will notice, we shall be leaving Heraklion on Thursday, April 13th instead of on Sunday, April 17th as originally planned.

Looking forward to our meeting, I am

Yours sincerely,

Chandra

S. Chandrasekhar

enclosures: 2 papers; itinerary

Iraklion 19-5-89

Prof. S. Chandrasekhar
LASR, The University of Chicago,
933 East 56th Str.,
Chicago, Ill. 60637,
U. S. A.

Dear Chandra,

Here is my first report on the $\delta=2$ solutions. It will be clear that the problem is very interested and quite different from what we have done in the previous solution ($\delta=1$).

(i) The derivation is explained clearly on pages 1-6 and it does not require any calculation, by using results from your "Kerr"-1978 paper and the expression for "$\mu_2 + \mu_3$" given in air "New singularity paper". As in the $\delta=1$ case, the vacuum solution is not, but the Einstein-Maxwell solution is, asymptotically flat.

(ii) The three parts of the axis are also good. With very simple calculations (essentially that e^f —> const. while e^v remains finite) given on pages 7-9 we see that the different parts are regular except from the deficits.

(iii)The mass of each "black hole" is -2/p. I think it is better to preserve the convention chosen in your "Kerr" paper and let p take negative values (p<-1). As I will mention later, we might have to impose even more restrictions on the parameters.

(iv) About the "point" ($n=1$, $\mu=1$): Here comes the fint substantial difference from $\delta=1$ black holes. It is shown on pages 12 + 13 that now e^v goes to zero like X^2, hence the norm of the timelike Killing field would go to zero on ($n=1$, $\mu=1$) like X^4, i.e. much faster than the extreme R-N blackholes. On the other hand, e^f —> to a regular function of v(=the angular variable near $n=1$, $\mu=1$). This would mean that

$$e^{-2v}\, e^f\, (n^2-\mu^2) \left[\frac{dn^2}{\Delta} + \frac{d\mu^2}{\delta} \right] \sim \frac{1}{X^3} \left[\frac{d\eta^2}{\Delta} + \frac{d\mu^2}{\delta} \right] .$$

Moreover, since $\Delta\delta \sim X^2$, even $d\varphi^2$ would get a term which would behave like

$x^{-2}(d\varphi)^2$ and it would require a new type of making the extension to negative values of x and demonstrating the smoothness of the surface $x = 0$.

(v) The δ=2 Einstein-Maxwell metric is too big for the computer in Crete, to evaluate the simplest curvature invariant $R^{ab} R_{ab}$ generally (since $R= g^{ab} R_{ab}=0$ by the Einstein-Maxwell equations). I immediately run into memory (not time) problems. After a lot of effort and splitting the problem to smaller ones I' ve managed to evaluate $R^{ab} R_{ab}$ for $q=\sqrt{3}$, $p=-2$, $\eta=2$ or $\eta=3$. I am pretty sure that these are generic numbers for the problem. The only denominators, which would represent curvature singularities, are $\beta\bar{\beta} - \alpha\bar{\alpha}$ and $(\beta-\alpha)(\bar{\beta}-\bar{\alpha})$. In fact, it is

$$R^{ab} R_{ab} \sim \frac{1}{(\beta\bar{\beta}-\alpha\bar{\alpha})^8 \left[(\beta-\alpha)(\bar{\beta}-\bar{\alpha})\right]^8} .$$

Since the axis is regular (curvature-wise), we knew that we should not worry about η^2-1 or $1-\mu^2$. However, the fact that $\eta^2-\mu^2$ does not appear in the denominator as well is a good indication that $n = \mu = 1$ might be a surface. So, the behavior of $R^{ab} R_{ab}$ is exactly as in the δ=1 case (c.f. Eq. 37).

(vi) On page 10 of the notes it is demonstrated that $\alpha\bar{\alpha}-\beta\bar{\beta}>0$ for the allowed range of η, μ, p, q. Hence, there is no problem from this term.

(vii) I am still not clear what is the situation with the vanishing of $K=(\beta-\alpha)\cdot(\bar{\beta}-\bar{\alpha})$. For some values of p and q I did find numerically roots of K for $\eta>1$, $-1<\mu<+1$. These would correspond to naked singularities. However, this is not the end of the story, because the solution does not have to be smooth for every value of the parameters. The real question is: Is there a range of the parameter q for which $K = (\beta-\alpha)(\bar{\beta}-\bar{\alpha})$ (given on page 10) has no roots for $\eta>1$, $-1<\mu<1$? This would exclude naked singularities. I do not know yet the answer to this question.

(viii) I anticipate two possible outcomes. Either the spacetime would be smooth (no naked singularities) either it would look like the figure for every value of q, where $\sim\!\sim$ is a curvature singularity.

In both cases I expect that the extension across $x = 0$ to be difficult to find it. Or, at least, it is a form of the metric near x =0 quite different from what we have experienced with.

(ix) Both cases would be interested, if we could demonstrate that the surface $\chi = 0$ (or the surface except a single meridian circle) is smooth. It would be a "black hole" in which the time translational Killing field becomes null on the "horizon" much faster ($\sim x^4$) than anything we know. In fact, if zero surface gravity is characterizing the extreme black holes (vanishing $\sim x^2$), these would be "superextreme" black holes.

(x) For q=0, p=±1 the metric is a cavuum solution and the part of the axis between the two points is a curvature singularity. The solution is asympto-tically flat.

The things I am planning to try next is

- Roots of $(\beta-\alpha)(\bar{\beta}-\bar{\alpha})$?
- Get a nicer treatement of $(\eta^2-\mu^2)\left[\Delta^{-1} d\eta^2+\delta^{-1} d\mu^2\right]$ and the form of the metric near $\chi=0$.
- If possible with my computer capabilities, try to get the behavior of a curvature scalar near $\chi=0$.

(xi) The experience with the δ=1 and δ=2 solutions very strongly suggest that the following general theorem might very well be true:

Consider any stationary, axisymmetric, asymptotically flat, vacuum space-time. This solution would have some representation interms of the Ernst potential $\mathcal{E}$ (for the conjugate variables). Then $\mathcal{E}$ and $\mathcal{E}^*$ would have some formal counter parts F and G, obtained by taking advantage of the presence of some parameters to perform a "realization" of $\mathcal{E}$. These (F, G) represent, via (X, Y), a new stationary axisymmetric vacuum solution which is not asymptotically flat. Apply the one-to-one correspondence to this (the second) vacuum solution and obtain the associated stationary axisymmetric Einstein-Maxwell electrovacuum solution. Claim: This second solutions is asymptotically flat!

It seems that the combination of these two "generation" techniques restores asymptotic flatness !!

I do not know how to prove the theorem generally, mainly because I do not know how to make precise the $(\mathcal{E}, \mathcal{E}^*) \longleftrightarrow (F, G)$ correspondence.

I plan to work some more examples to check its validity, preferably outside the T-S type solutions.

<u>About summer plans</u>: A school on Computer Algebra and applications to relativity is taking place in Brazil July 24th-August 11, that I would like to attend. If this is materialized, I would have to leave Chicago on July 23rd, which leaves little time for the things we are planning to do (The Newton proofs and your Selected Works). I would like to check with your schedule Ju-

ne 20-30. If it is O.K. with you, I could possibly come around June 24-25 and get the extra week in Chicago before the Colorado meeting.

I still have to receive your letter-invitation, to get the required visa. If it is not already in the mail, could you please send it by FAX, it could save me an unnecessary trip to Athens.

I hope you had a good meeting at Yale. With best regards, to Lalitha as well,

Yours,

Basilis