Iraklion, July 3, 1988.

Dear Chandra:

Here are my remarks on you paper on the two black holes solution. I am refering to them in the order they appear, not order of seriousness.

1. Page 3, line 7, it should be $\sqrt{G}\, M_i = Q_i$, the square root is missing.

2. Page 7, Eq. 18: I would had written $\eta^2 - 1$ and $1-\mu^2$ <u>before</u> $(e^{-\nu})_{,\eta}, \cdots$.

3. I find too large the computational-conceptual gap before arriving in equation (44). An intermediate equation would have been useful.

4. Page 14, first line of section 4. I would remove the word "necessary". I agree that it is important but I would not... commit suicide if the energy flux weren't there.

5. Page 17, Eq. (54), it should be set $= 0$ (at the end).

6. Pages 17 and 18.
I thing you should define Eq. (58) as

$-E_\mu =$ R.H.S. of eq. (58), in order for Equations 59 and 61 to read

$$\frac{1}{\eta^2-\mu^2}\left(\frac{\partial E_\eta}{\partial \eta} - \frac{\partial E_\mu}{\partial \mu}\right) = 0$$ and

$$\int\left(\frac{\partial E_\eta}{\partial \eta} - \frac{\partial E_\mu}{\partial \mu}\right) d\eta\, d\mu = \int (E_\eta\, d\mu + E_\mu\, d\eta)$$

since, as I understand it, your ultimate purpose is to arrive at the line interpral $\int (E_\eta\, d\mu + E_\mu\, d\eta)$.

7. Page 18, Eq. (60), I get

$$d\eta\, d\mu\, d\varphi \sqrt{g} = \cdots = + (\eta^2-\mu^2)\, d\eta\, d\mu\, d\varphi .$$

8. Page 19, Eq. 63, last line should be

$$+ \frac{1}{V^2}\left(V_{,\eta} X_{,\mu} - V_{,\mu} X_{,\eta}\right) = 0$$

9. Page 20, Eq. 66, you should include $+ O(\eta^{-1}) = 0$.

10. Page 20, Eq. 69 should read

$$\mathcal{D} y = \frac{\mu_1}{\eta^2} y$$

11. Page 21, Eq. (71), more correctly should be written as

$$\mathcal{D} \exp\{i\sigma[\cdots]\} = 4i\sigma \frac{(\mu_1+\mu_2)^2}{\eta^3} \exp\{i\sigma[\cdots]\} + O(\eta^{-3})$$

12. Page 21, Eqs (72) and (73): Equation (70) depends only on σ^2, not on σ individually. Hence, I am surprised that the solution described by equations (72) and (73) is not symmetrical in $+i\sigma$ and $-i\sigma$.

However, from the solution (72)-(73) we obtain - I am now talking about equation 74 -

$$y = \exp\{+ i\sigma[\cdots]\},$$

not $\pm$.

16. Page 24, Eq. 83. the brackets $[\![$, $]\!]$ should be removed. Also the ...

13. Page 21, two lines after Eq. (74), the expression should read

$$\sigma^2\left[1+2(\mu_1+\mu_2)^2\right],$$

remove all "=" sign.

14. Page 23, Eq. 79, and others in this section. You keep terms which obviously are of higher order. For instance, the term $-4\frac{(\mu_1-\mu_2)}{\eta^2}X_{,\mu}$ in the second line of eq. (79) plays no role and it can only confuse the reader.

15. Page 23, Eq. 80 should be $X=\eta Z$.

16. Page 24, Eq. 83. the brackets $[\![\;,\;]\!]$ should be removed. Also, the term in $[\quad]$ should read $2\sigma^2(\mu_1+\mu_2)^2 + \lambda_2$.

17. Page 25, Eq. (89). It is not clear to me that the expression you are giving is the most general. Probably you should set

$$X \to \eta\sum\left(S_n e^{+i\sigma[\dots]} + \tilde{S}_n e^{-i\sigma[\dots]}\right) e_n^{-3/2}(\mu),$$

i.e. I think that you should use different coefficients (possibly, complex conjugate) for the plus and the minus phases. The same remarks apply to Eq. (90).

18. Page 26, Eq. 92, the second term, the subscript y is missing in the bracket.

19. Page 27, Eq. 99. You should be using the $\sim$ sign, instead of the equality sign:

$$U \sim \frac{\mu_1}{\xi+3}, \quad U_{,y} = -V_{,t} \sim -\frac{\mu_1}{(\xi+3)^2}$$

20. Page 28, Eq. 101 the third term in the first line should be $-(X_{,y3}+X_{,3})$, i.e. the 2 should be removed.

21. Page 29, first line should be:

is minus the cosine - - - - .

Also the second sentence should be:

"We change from (x,y) to (x,r) coordinates, hence".

22 Page 30. Here is my serious comment which, if corred and accepted, should change considerably the remaining of the paper.

The lemma should be

$$\left(x \frac{d^2}{dx^2} + 2\frac{d}{dx} + \frac{\sigma^2 M_1^4}{4x^3}\right)\left(e^{\frac{i\sigma M_1^2}{2x}}\right) = 0.$$

(an equality, not to some order).

Then equations (117) and (118) should be

$$(1-r^2) f_{,rr} - 4f - \frac{1}{M_1} g_{,r} = 0,$$

$$(1-r^2) g_{,rr} + 2r g_{,r} + 4M_1 \left[(1-r^2) f_{,r} + 2rf\right] = 0.$$

The general solution of these equations is

$$f = \frac{\alpha r^2}{2M_1} + \beta r + \gamma,$$

$$g = \alpha r (1-r^2) - 2M_1 r(\beta r + 2\gamma) - \delta,$$

where $\alpha, \beta, \gamma, \delta$ are constants.

23. If my remark # 6 is accepted, then Eq. (165) should be

$$-E_3 = \text{R.H.S. of Eq. (165)}.$$

24. Page 40, second line from top. Instead of "positive exponent" I would say "exponent with the plus sign". Imaginary numbers are not positive or negative.

25. In the REFERENCES.
You should include references to Whittaker (1937), and to Chandrasekhar and Ferrari (1989), they are referred in the text.

26. Second page in the Appendix, Eq. (A8), there is one extra parenthesis (c) and the last term should be

$$\dot{q}^2 \sinh\psi \cosh\psi \sin^2\theta.$$

The main thing is your reaction to my remark # 22.

I am sending this FAX.

Persides just arrived in Iraklion. He will stay now for one day and he will be back, staying for one month, July 15 – August 15.

I hope you had good time in Germany.

With best regards, to Lolitha as well.

Sincerely yours,

Basilis.

Iraklion, July 5, 1988.

Dear Chandra:

This is my second letter on your paper on the perturbations of the two black holes solution, after our conversation on July 4. I could not send my previous letter because our FAX is down (the machine is O.K., the telephone line is disconnected, and my feeling is that it will be connected again one of these days).

Although I do not know how to derive them, the attitude now is that equations (117) and (118) should be taken as correct. Remarks:

(1) Page 32, Equations (129) and (130), the square roots should be

$$[(\lambda_1-\lambda_2)^2+16(\lambda_1-2)]$$

The same correction should be made in the second of Eqs. (132).

(2) I do not see the need to go through the arguments presented on page 33, i.e. to define $k_\pm = m \pm n$ and then, quite artificially to set

$m = \ell^2$, $n = \ell$ (Eq. 134). Here is a simpler, and conceptually correct and complete, presentation:

We ~~sho~~ solve Eq. 127 by power series. The recursion relation is

$$(n+1)(n+2)\, a_{n+2} = [n(n-1) - k_\pm]\, a_n .$$

Hence for the equation to admit polynomial solutions the series should terminate, i.e. we should have

$$k_\pm = \ell(\ell-1), \quad \ell \text{ integer positive.}$$

with $\ell \to \ell+1$, we get the second choice, $\ell(\ell+1)$.

(3) Page 33, Eqs. (133), the square roots should be

$$\sqrt{n^2 - 4m + 12}$$

(4) ~~Page 34, Eq. (138) I think the correct is~~

$$\cancel{A_\pm = \frac{1}{2}\left[2 + i\sqrt{3\ell^2-12} \mp \sqrt{\ell^2 - 8i\sqrt{3\ell^2-12}} \right] B_\pm}$$

~~Accordingly, Equations 139, 140, 141, 142 should be modified as described in the following page. Equations (144) and (147) also should be modified.~~

I WAS WRONG. Eq. 139, 140, 141, 142, 144, 147 are correct.
Page 13 is not sent.

(5) Page 35, Eq. (148). I do not see why $C_2^{-1/2} = 0$. (I agree that $C_3^{-3/2} = 0$).

~~Also $B_4 \neq 0$.~~

(6) Page 36, second of Eqs. 149, it should be $\cot \delta_{\ell+1} = A_{\ell+1} / B_{\ell+1}$. Then the remaining expressions are correct.

(7) Page 37, Second of eqs. 159 should be $2M_1 f = \cdots$.

(8) Page 43, Eq. 178, the second term should be

$$+ \delta'_m f(\delta_n) C_m^{-1/2}(v) .$$

(9) Page 43, Eq. 180, in the second line, the argument of the Real Part should be

$$\delta'_{m+1} \delta'^{*}_n f(\delta_{m+1}) f^{*}(\underline{\delta_n})$$

The scattering problem you are formulating seems correct to me.

(10) Two more corrections on the geodesics.

~~Page~~ Last page, the second term in Eq. A21 should be

$$+\frac{1}{U^2}\left(\frac{\partial U^2}{\partial \psi}\dot{\vartheta}-\frac{\partial U^2}{\partial \vartheta}\dot{\psi}\right)\dot{\vartheta}.$$

Similarly, the second term in Eq. A22 should be

$$-\frac{1}{U^2}\left(\frac{\partial U^2}{\partial \psi}\dot{\vartheta}-\frac{\partial U^2}{\partial \vartheta}\dot{\psi}\right)\dot{\psi}.$$

The term E^2 should not be in neither equation.

I hope, I should be able to send both letters tomorrow, by FAX.

Best regards,

Yours Sincerely,

Basilis.