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Laboratory for Astrophysics and Space Research

March 16, 1987

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Dear Basilis,

Your letter of March 7th and enclosures arrived a few days after I had communicated our paper to the Royal Society. I have not read your calculations on 'perfect fluid in special relativity' nor have I read in detail your list of corrections. From our telephone conversations I had noted what they were and checked them all for myself (incidentally, some of the errors were transcription errors from my notes). But there are two points which stand out in your letter on which I did make some comments on the telephone. Let me repeat them.

1) With regard to my reluctance to using your terminology "two pieces of null dust":

If one did start defining the null dust with the expression $T^{ij} = E k^i k^j$, then the evaluation of $T^{ij}_{;j}$ will proceed exactly as in equations (15), (16), and (17) of Proc. Roy. Soc. Lond. A403, 189; and we shall find that we have to distinguish the two cases: $k_{(0)} = +k_{(3)}$ and $k_{(0)} = -k_{(3)}$. And the two cases will lead to the same final results, namely, that $(U^u)^2 = -N_{,v}$ and $(U^v)^2 = \bar{N}_{,u}$. Instead of constantly distinguishing the two cases, one can treat the problem in a unified way by letting ab initio that $\frac{1}{2} T^{ij} = \epsilon k^i k^j + \bar{\epsilon} \bar{k}^i \bar{k}^j$ as we have done. In other words, the fact that under the circumstances considered null dust can be regarded as consisting of two sorts of massless particles is a deduction, not a postulate.

2) You have objected to the use of the words "impenetrable barrier" and the simple "trap" in the last paragraph of section 6. The choice of the words was deliberate. In the case of null dust, no particles can emerge beyond $u^2 + v^2 = 1$ by the presence of a singularity in all cases. In constrast, the ($\epsilon = p$)-fluid is trapped even when the singularity has disappeared by the motions having become vortical. In the later case,

[illegible]!) This can be incorporated when the paper is "accepted"!

the contrast with special relativity is even greater because of the coupling between the $\mathcal{N}$ - and $\mathcal{S}$ -variables in general relativity; a coupling absent in special relativity. The use of the word "radically different" on top of page 43 is not with respect to null dust as against the ($\epsilon = p$)-fluid but rather as against special relativity and general relativity. In the two cases illustrated in Figures 1a and 1b, the particles emerge as null dust in special relativity, ~~in all cases~~. In general relativity, they simply do not emerge even though the underlying causes are different for null dust and for the ($\epsilon = p$)-fluid.

As I told you, my reason for including this section is not for providing a "seed for later investigations" but rather to emphasize, that the transformation of null dust into an ($\epsilon = p$)-fluid occurs both in special and in general relativity for the purposes of the conjecture in the concluding remarks; and that what actually takes place in general relativity cannot be inferred from the simple examples considered by Penrose in special relativity.

By the same mail I have also written to the Commission of the European Communities and to Professor H. Sato at the Research Institute for Fundamental Physics in Kyoto.

With best wishes,

Yours sincerely,

Chandra

S. Chandrasekhar

P.S. I have just noticed that in eq. (132) I have incorrectly written $\sqrt{s}$ in place of $s^{-1/2}$. This & other corrections you will (undoubtedly!) find can be incorporated when the paper is 'accepted'!

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March 17, 1987

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Dear Basilis,

This is in continuation of my letter of Monday. On reading over and thinking once again of the parts describing the principal results (pages 31, 39, and 42-45), I am now inclined to delete the qualifying words "vortex" and "vortical": on page 43, line 7 from the top, and on page 45, line 10 from the top; replace them by "cloud" and "clouds" on page 43, and "isolated" on page 45. I realized the need to make these changes after deriving the following formula for $(U_3)^2$:

$$(\delta - \Delta)\left[(\phi_{,r})^2 - (\phi_{,s})^2\right](U_3)^2 = \left\{\left[u\sqrt{1-u^2} - v\sqrt{1-v^2}\right]\phi_{,r} - \left[u\sqrt{1-u^2} + v\sqrt{1-v^2}\right]\phi_{,s}\right\}^2$$

Since $\phi_{,r}$ is finite for $s \to 0$ while $\phi_{,s} \to 0$ for $s \to 0$, it follows that $(U_3)^2$ always tends to 0 when $s \to 0$, independently of whether $s = 0$ is a singularity or not. I believe, ~~that~~ this fact that I had not appreciated before (but you might have), is sufficiently important to include in the paper somewhere: perhaps after equation (88) -- and draw attention to it again on line 4, page 32.

With best wishes,

Yours sincerely,

Chandra

S. Chandrasekhar

Iraklion, March 29, 1987.

Dear Chandra:

I want to mention a few minor corrections on our paper and to comment on your letters of March 16 and March 17. I have not received the pages with the conjenctures deleted but I do not want to keep my reply any longer, waiting for them.

Corrections on the paper:

(1) Page 2, line 3.
1985 a, b, c, 1986 a, b, ...
"c" should be deleted.

(2) Page 8, last line, just before equation (31).
Papers I, II, and III.
Either we write IIa, IIb or we delete II.

(3) Page 32, the second line of equation (119) should read

$$= \frac{v}{2(1-2pv+v^2)\sqrt{(1-v^2)}}\, e^{-f(0,v)}\, f_{,v}(0,v)$$

(4) Page 35, Eqs (132), as you have mentioned in your subsequent letter, it should be $s^{-1/2}$.

(5) Page 36, Eq. 135, I suggest

$(-1 \le x \le 1, \; -1 \le y \le 1, \; 0 \le x+y \le 2)$,

since only the first two inequalities suggest that $x+y$ could become as little as -2.

(6) Page 37, the first two of equations (138):

$$\tilde{\Psi}_0 \sim -\tilde{\Phi}_{00} \sim ----$$
$$\tilde{\Psi}_4 \sim -\tilde{\Phi}_{22} \sim ----,$$

since they are actually behaviors and not limits.

(7) Page 39, last 1½ line.
Either ; should changed into , (comma), or the text after the semicolon should be modified.

(8) Page 41, Eqns. (148) should be

$$V^u = V^0 + V^3 = -2\phi_{,v}, \quad V^v = V^0 - V^3 = +2\phi_{,u}.$$

The justification is the following calculations:

$$V^u = V^i \frac{\partial u}{\partial x^i} = V^0 \frac{\partial u}{\partial x^0} + V^3 \frac{\partial u}{\partial x^3} = V^0 + V^3 = \phi_{,3} - \phi_{,0} = -2\phi_{,v}$$

$$V^v = V^i \frac{\partial v}{\partial x^i} = V^0 \frac{\partial v}{\partial x^0} + V^3 \frac{\partial v}{\partial x^3} = V^0 - V^3 = \phi_{,3} + \phi_{,0} = 2\phi_{,u}$$

(9) Page 41, equations (150)–(151) and text in between. I suggest that we use the notation

$$\phi(u,v) = \bar{\Phi}(u) + \Phi(v) \qquad (150)$$

where $\bar{\Phi}(u)$ and $\Phi(v)$ are - - - ..

$$V^u = -2\Phi_{,v} \quad \text{and} \quad V^v = 2\bar{\Phi}_{,u} \qquad (151)$$

It is always our policy to use (F, f) for the metric functions and (Φ, ϕ) for the potentials. Lets follow it here as well. But we definitely need the "2"'s in equations (151).

(10) Page 43, line 8 from the top, it should be

, as $u^2+v^2 \to 1-$

(Since $u^2+v^2<1$, $u^2+v^2 \to 1-0$.)

(11) Page 46, Reference 3, it should be

223–259.

I also suggest that we include

(paper I)

(paper IIa)

(paper IIb)

(paper III)

in references 3, 4, 5, 6 respectively.

(12) Page 48. Somewhere around Eqs. (A4)–(A6) I suggest that we mention that

χ, q_2 refer to the electromagnetic solution but we suppress the subscripts "e" used in the text. I think that we need such a statement because there is interplay in this section between gravitational and electromagnetic solutions.

(13) Page 49, Eq. (A12): We should suppress the subscript **e** from q_2.

(14) Page 57, text before eq. (B31). I suggest

; and from the behaviour (B29) of v_e^2 - - -

About the two letters.

I agree with everything. I have verified independently equation

$$(\delta-\Delta)\left[(\phi_{,r})^2-(\phi_{,s})^2\right](U_3)^2=\left\{\left[u\sqrt{1-u^2}-v\sqrt{1-v^2}\right]\phi_{,r}-\left[u\sqrt{1-u^2}+v\sqrt{1-v^2}\right]\phi_{,s}\right\}^2$$

and I think that the conclusion that $(U_3)^2 \to 0$ when $s \to 0$ is very interesting (No, I did not know that).

My only point is that $U_3 = U_{(3)}$ is in fact the tetrad component of the velocity and that its vanishing may be due to the choice for the chosen tetrad. Fortunately, this is not the case here. In the enclosed calculations (pages 1 and 2) I demonstrate that the ~~covariant~~ ~~ten~~ contravariant tetrad component (which describes motion) goes to zero like $s^{\frac{k}{2}+1}$

(the presence of the fluid actually helps, since $k = k(r) > 0$).

Note, however, that when $q = 0$,

$$X = (1-\eta)^2 \sim \frac{s^4}{4(1-r^2)^2} \implies$$

contravariant, ~~tensor~~ tensor $u^3 \sim -2r\sqrt{A(r)}\, s^{\frac{k}{2}-1}$

and it would tend to zero or it would blow up depending on whether $k > 2$ or $k < 2$.

For a very tiny fluid, $u^3 \to \infty$, but this would happen only in those cases in which the purely gravitational solution is singular for $s = 0$.

For the Nutku–Halil solution, on the other hand, the claim seems to be that always, for $q \neq 0$ and for $q = 0$, contravariant, tensor $u^3 \to 0$.

Indeed. Now in the expression

$$u^3 = -\frac{r}{1-r^2}\,\frac{\sqrt{A(r)}}{\sqrt{X}}\, s^{\frac{k}{2}+1}$$

X should be substituted by

$$e^{(\nu+\mu_3)_{\text{Nutku-Halil}}}\sqrt{\Delta} = \frac{1-p^2\eta^2-q^2\mu^2}{(\Delta\delta)^{1/4}} \sim \begin{cases} \dfrac{q^2(1-r^2)}{s^{1/2}} & \text{for } q \neq 0 \\[2mm] \dfrac{s^{7/4}}{1-r^2} & \text{for } q = \end{cases}$$

Hence, we find that

$$u^3 \sim \begin{cases} -\dfrac{r}{1-r^2}\,\dfrac{\sqrt{A(r)}}{q\sqrt{1-r^2}}\, s^{\frac{k}{2}+\frac{5}{4}} & \text{for } q \neq 0, \\[3mm] -\dfrac{r}{\sqrt{1-r^2}}\sqrt{A(r)}\, s^{\frac{k}{2}+\frac{1}{8}} & \text{for } q = 0. \end{cases}$$

In both cases, $u^3 \to 0$, as $s \to 0$.

Do, all these, suggest any general scene to you?

With best regards,

Yours Sincerely,

Basilis.