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Laboratory for Astrophysics and Space Research

November 1, 1982

Dr. B. Xanthopoulos
University of Crete
Department of Physics
P.O. Box 470
Iraklion, Crete
GREECE

Dear Basilis,

Thank you for your letter of October 14th sent to the Royal Society. I saw it only on Friday, October 22, after my lectures at Cambridge.

At the Royal Society, I also got a copy of the referee's report on your paper. On my return I discussed the report with Bob Geroch; and I agree with the comments (copy enclosed) that he made for me. I think that the 'substantive comment' which Bob suggests could be added at the end of the introduction on page 5; and the minor corrections are of course easily made.

My recommendation would be that you return your manuscript to Evans with the suggested revisions. Also, in a covering letter you might add that you have made the corrections after discussions with Bob Geroch and myself and that we approve the manuscript as submitted; and also that the revisions were necessary for a proper setting of the problem (as a matter of politeness!).

I see from your letter of October 14 that the people at Crete are keeping you very busy. Sotirios, in a letter, did tell me that your lecturing assignments at Crete would be very heavy; and that for that reason you should have stayed on at Salonika even in a somewhat inferior position. I am sure you have discussed the matter with Sotirios and decided to go to Crete anyway.

All the page proofs of my book are in. I hope to return all of them next week. Then there will be some three weeks' work on the index. And that will be the end; and how I look forward to it!

I was astonished that I was asked to give one of the invited addresses at the GR 10 at Venice. And when I expressed some hesitation, Bertotti thought that I should give the lecture since there might be no future occasion! It will be some compensation if we can meet at Venice.

Yours sincerely,

Chandra

S. Chandrasekhar

enclosures: 2

Comments on referee's report of "Local Toroidal Black-Holes that are Static and Axisymmetric".

Robert Geroch
Oct 26, 1982

Let me begin with the smaller points at the end of the report:

pp 2, line 6 from bottom: I agree with the referee. One could, e.g., change "null hypersurface" to "spacelike surface".
pp 10, end of Sect 3: I guess I would agree that it is just marginally confusing. One could leave as is, or, e.g., change the second to last sentence to read "...with the flat metric..."; and the last sentence to "Flatness of the metric (3.12) also follows directly from the results of section 5."
pp 12, before 4.14: The first sentence in the report refers to the omission of the word "metric". The second sentence, I assume, refers to Eqn. (4.16).
pp 13, middle: He proposes, I take it, that "with" be replaced by "as".

Now to the referee's conceptual point. This is essentially the point which was raised, you may remember, in the seminar on this work. Then, it was part of a discussion of the extent to which these toroidal holes are "physically interesting". The point, as I understand it, is the following. It is true that these toroidal local holes can always be realized as objects in a space-time. That is, one can find a distribution of external matter for them such that the final space-time is asymptotically flat and nonsingular outside the horizon. But - and this is the key point - there is a theorem of Hawking to the effect that any such matter distribution must violate an energy condition somewhere outside the horizon (i.e., it must have negative energy density somewhere). Thus, there is at least a certain sense in which these objects cannot be expected to arise astrophysically.
On rereading "Distorted Black Holes", I realize that the point is not made there as sharply as I would now have wished. [See last sentence of last full paragraph on pp 682; last sentence of first paragraph ending in the second column of pp 683; second sentence, first full paggraph, second column, pp 689.]

The referee's point, I gather, is that this issue should somewhere be made explicit in the paper. I would agree. My suggestion would be to add a short, few-sentence, paragraph in the introduction, which refers to and summarizes in words the theorem of Hawking, and then expresses an appropriate degree of concern about the physical significance of these holes.

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Laboratory for Astrophysics and Space Research

Mr. W. G. Evans, Editor
The Royal Society
6 Carlton House Terrace
London SW1Y 5AG
ENGLAND

Dear Mr. Evans,

I am communicating herewith a paper entitled "Local toroidal black-holes that are static and axisymmetric" by B. C. Xanthopoulos for consideration towards publication in the Proceedings of the Royal Society.

I might add that I have read and scrutinized the paper very carefully. Besides, my colleague, Professor Robert Geroch, whose work the present paper extends, has also scrutinized the paper and strongly approves its publication. As for referees, Roger Penrose might be the most discriminating; but Brandon Carter and Stephen Hawking are equally competent to evaluate it (if they have the time!).

I should be grateful if further communications relative to this paper are sent to me directly with copies to the author at Crete.

With thanks for your consideration,

Yours sincerely,

S. Chandrasekhar

S. Chandrasekhar

enclosure

From the office of
S. CHANDRASEKHAR

Basil's

I am sending a duplicate of the set I sent you to Rhoden. Also the 11th Copy of Chap VI - 50 pages. Since the entire book has to go to press on Jan 8, I shall call you on Thursday Jan 7 for corrections to what I am sending today. Thanks.

Chandra,

January 11, 1982.

Dear Chandra:

A have a few remarks on the final version of Chapter XI, pages 1-50.

1. Page 6, second term of the first line of equation 16, the term $V_{,3}$ is missing inside the bracket.

2. Page 8, second line from the end, Probably it would have been more correct to write "we can impose a coordinate conditon..." instead of "we have to".

3. Page 12, Equation (42), in the second term you should change Δ to δ.

4. Page 18, line 5, also page 16, first line, the correct is "Gürses and Xanthopoulos have...".

5. Page 27, equation 104, they should be $\tilde{A}$ and $\tilde{B}$ the tildes are missing.

6. Page 28, The last equation of equations (113), you need $e^{2B} = e^{2(\psi+\nu)} = \Delta\delta$.

7. Page 29. Just before equations (116) you could end that you are also using that $H = A + iB$. It helps enormously.

8. Page 37, In the right hand sides of the last two equations of equations (135) the denominatos should be $(\bar{p}^*)^3$ instead of $(\bar{p}^*)^2$.

9. Page 39, Eq. (146): The last term in the left-hand-side should be $-6ic\bar{p}$ and the

last term in the right-hand-side should be $-\ell_2 \frac{5\bar{p}^4}{\bar{p}^2}$.

10. Page 41. At the end of the big paragraph I suggest that you explain what you mean by a static distorted black hole. I am suggesting something like: A Weyl solution (or a static axisymmetric solution) is a static distorted black hole if it has an extension to a ~~true~~ true static black hole such that the original Weyl solution forms a neighborhood of the horizon.

~~11. Page 42, top. You should mention that the form (4.4) represents the general static axisymmetric metric.~~

11. Page 42, line 7 from the end. The horizon is at $r=2M$, the 2 is missing.

12. Pages 43–46. You have omitted the constant M in a few places. The correct equations are:

(155): $\Psi = \chi\sqrt{\Delta\delta} = M\chi\left[(\eta^2-1)(1-\mu^2)\right]^{1/2}$

(158): $\chi = \chi_s = \frac{1}{M}\cdot\left[\frac{\eta-1}{(\eta+1)^3(1-\mu^2)}\right]^{1/2}$

(164): $\chi = e^{-\psi+\nu} = \chi_s e^{s'} = \frac{1}{M}\left[\frac{\eta-1}{(\eta+1)^3(1-\mu^2)}\right]^{1/2}\cdot e^{s'}$

(165): $e^{\beta} = e^{\psi+\nu} = M\left[(\eta^2-1)(1-\mu^2)\right]^{1/2}$

(166): $e^{2\psi} = M^2 (1-\mu^2)(n+1)^2 e^{-S'}$

(167): $ds^2 = \frac{n-1}{n+1} e^{S} (dt)^2 - M^2 (1-\mu^2)(n+1)^2 e^{-S'} (d\varphi)^2 -$

$$- \frac{e^{h_2+h_3}}{M\sqrt{n^2-1}} \left[(dn)^2 + (n^2-1)(d\theta)^2 \right]$$

13. Page 45, Eq. 163 and follows: The symbol $S' = S'(\eta, \mu)$ is not a good choice. You have the subscript "S", which stands for Schwarzschild, and S indicates all the additional corrections. I am suggesting that you either change S' to something else or you always write $\delta S'(\eta, \mu)$.

14. Page 46. The first term of equation (168) should be $-\frac{\mu}{1-\mu^2}(h_2 + h_3)_{,n}$

15. Page 47. The second and the third terms in the second line of equation (171) should be combined to the single term $+\frac{4\eta}{\eta^2-1} S'_{,\mu}$.

16. Page 50, Equation (181). Since the Legendre polynomials are normalized so that $P_n(1) = 1$, equation (181) could be written $\sum_{n=0}^{\infty} A_{2n+1} = 0$.

17. I understand and I agree with the condition of local flatness but I do not think that the presentation is clear. I am suggesting the following: Just before equation (172) you

write the condition as $\lim_{\theta \to 0,\pi} \frac{\sin^2\theta\, g_{\theta\theta}}{g_{44}} = 1$

or for the metric written in the form (167)

$$\lim_{\mu^2 \to 1} \frac{e^{\lambda_2+\lambda_3}\sqrt{n^2-1}}{M^3 (n+1)^2 e^{-s}} = 1. \qquad (172)$$

(Equation 172 is not corred, the terms $(d\theta)^2$ and $\sin^2\theta$ should not be there).

Next we set

$$e^{\sigma} = \frac{1}{M^3}\left[\frac{n-1}{(n+1)^3}\right]^{1/2} \cdot e^{\lambda_2+\lambda_3+s}$$

(i.e., equation (173) is defined everywhere and not only for the limit $\mu^2 \to 1$). Then the condition (172) expressed in terms of σ becomes

$$\lim_{\mu^2 \to 1} \sigma = 0, \quad \forall \; n \geq 1.$$

Equations (174) and (175) are corred, and they are not affected by the inclusion of the factor M^3. The corred final form (182) of the metric is

$$ds^2 = \frac{n-1}{n+1} e^{s}(dt)^2 - M^2 e^{\sigma-s} \frac{n+1}{n-1}(dn)^2 - M^2 (n+1)^2 e^{-s}\left[(1-\mu^2)(d\psi)^2 + e^{\sigma}(d\theta)^2\right.$$

$$= \frac{n-1}{n+1} \cdot e^{s}(dt)^2 - M^2 e^{\sigma-s} \frac{(n+1)}{(n-1)}(dn)^2 -$$

$$- M^2 (n+1)^2 e^{-s}\left[(1-\mu^2)(d\psi)^2 + \frac{e^{\sigma}}{(1-\mu^2)}(d\mu)^2\right].$$

The constant M can be absorbed by a redefinition of t.

This concludes my remarks on chapter XI.

Considering static, axisymmetric black holes with spherical topology I was able to obtain the same conditions as you (namely, to exclude the Legendre functions of the second kind and to obtain that $\sum A_{2n+1} = 0$) by demanding that the determinants g_2 and g_4 of the ~~two-dimensional~~ metrics on the horizon and of the four-dimensional metrics are finite and non-zero for $x \to 1$. This is the idea which was used in the last two letters of mine about the toroidal black holes. So the idea which works for spherical black holes seems to exclude all the toroidal black holes. I will write on that in a few days, I don't want to delay material related to the book.

Today, after 23 days, I received your letter and your ~~words~~ calculations, mailled on December 21st!! And the nth draft of the eleventh chapter two days ago.

Best regards to your wife.

Sincerely,

Basilis.