

THE UNIVERSITY OF CHICAGO  
THE ENRICO FERMI INSTITUTE

933 EAST 56TH STREET  
CHICAGO • ILLINOIS 60637  
AREA CODE 312-753-8507

Laboratory for Astrophysics  
and Space Research

December 1, 1980

Dr. Basilis Xanthopoulos  
University of Thessaloniki  
Department of Astronomy  
Thessaloniki, Greece

Dear Basilis:

I am enclosing the rest of the nth draft of Chapter 4. As you will notice I have added an entirely new section 32 which answers some of your specific questions concerning the way my original section 32 was written. While I should like to wait for your comments on the final draft (which is substantially different from the (n-1) copy), I think I shall proceed with the typing of this manuscript. However, I will not be filling in the formulae for another two weeks. Perhaps I shall call you on December 15 to find out if there is anything serious that I ought to change.

I am starting this afternoon to complete my notes for Chapter 5 and hopefully I should be able to send you the (n-1) draft of this chapter in perhaps three weeks. Then, at long last, I shall start on Chapter 6; and if all goes well I can contemplate coming to see you during the first week of February. I should <sup>have</sup> wished to come at least a week earlier but I have a lecture to give in Stanford towards the end of January.

There seems to be one matter on which we seem to disagree. You, along with Detweiler, do not seem to think much of my transformation theory. I personally feel differently. Time will tell!

With best wishes,

Yours sincerely,



S. Chandrasekhar

SC:gw

Enclosure

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February 4, 1980

Dr. Basilis Xanthopoulos  
University of Thessaloniki  
Department of Astronomy  
Thessaloniki, Greece

Dear Basilis:

Herewith the rest of my notes for Chapter VI. (The 13 odd pages that were included in the last batch was by mistake: they were the original notes I had made after your explaining to me all that two years ago.) The present batch includes the same material somewhat revised.

The only parts of my notes on Chapter VI, with which you may not be familiar, are those relating to the components of the Riemann tensor (pp. 33-76) and the proof of the Carter-Robinson theorem (pp. 91-102). I am also including a tentative contents of Chapter VI. The parts of it I am particularly anxious to talk to you about is the extent to which I should include the general discussion I propose in section 54.

There are two matters connected with the Kerr metric I want to learn from you. They concern the Killing tensor and the Killing spinor and their use in following the propagation of the polarization vector along null geodesics. The second matter pertains to relating the constant in the asymptotic behaviour of  $\omega$  with the angular momentum of the source.

I am including a xerox of three relevant papers which I want to discuss with you.

I shall start today on writing my nth copy of Chapter V. I am three days behind my schedule so I may not be able to send the xerox of it before February 11. I will send it Express so you may have a chance to look at it before we arrive.

There is so much to discuss that I am afraid that we shall have to work quite hard during the five days I shall be there.

Yours sincerely,



S. Chandrasekhar

SC:gw

Enclosures

(1)

Thessaloniki, December 5, 1980.

Dear Chandra:

Here are a few minor remarks on the first 80 pages of the  $n^{\text{th}}$  copy of chapter IV.

- Page 2, line 8, you might consider to substitute the word "inner" with "inter".
- Page 3, line 9 from end, you write "model" instead of "mode".
- Page 7, the far end of the first line for  $G_{22}$  is not completed but it might be due to the zero.
- Page 11, line 8. You expand in  $e^{i\omega t}$  while the peoples Fourier analysis is  $e^{-i\omega t}$ . So, you might write "Fourier component with the frequency  $-\omega$ ".
- Page 13, in the second of equations (29) I would have written  $\Lambda^2 = \Lambda_+ \Lambda_- = \frac{d^2}{d\alpha^2} + \omega^2$ .
- Page 14, line 8. The relevant citations are 4 f, 8, b and j.
- Pages 15 and 16, equations 39 and 42, you write  $d\psi$  instead of  $\delta\psi$ .
- Page 17, last four lines. By reading the last paragraph I would immediately conclude that eq. (35) does not separate with the substitutions 35-39. I think it should be rephrased. Also, you

should state that the terms in  $P_e$  vanish by virtue of the previous equations.

- Page 24. In the title of section 25 I think that you should substitute the word "relating" with "related". Also, in line 11 from end the citation should be eqs (52) - (54).
- Page 25, first line. I would suggest starting "In principle the reducibility ...". I would certainly avoid the use of the word explicit in this sentence.
- Page 26, after eqns. (81) you might write "respectively".
- Page 30, eq. (99). I think that both  $L$ 's should be  $L^+$ .
- Page 32, first line, you need a "+" between the two top determinants.
- Page 32, last determinant in eq. (105), the  $\sigma_j$  in the right bottom of the determinant should be  $\sqrt{2}$ .
- Page 37, the citation before eq. (121) should be "116".
- Page 38, sentence after eq. 126. Use either solutions and were or solution and was.
- Page 53, In equations 177 and 178 you might use just lim, which is standard terminology.

- Page 65, first three lines. I think that something like "they emerged from the interplay of the two treatments" would be more clear.

- Page 69, Eq. 242, "l" is missing.

- Page 76, line 8, You need to cite equations 237 - 242.

I am waiting now for the rest of your notes.

We had some discussions about the possibility of evaluating the "missing integral"  $\Phi$ . I don't think that I can evaluate it even in the flat space limit, where for  $Z$  satisfying

$$\left[ \frac{d^2}{dx^2} + \sigma^2 - \frac{l(l+1)}{r^2} \right] Z = 0, \text{ we would like}$$

to evaluate  $\Phi = \int \frac{Z}{x} dx$ .

Best regards.

Sincerely,  
Basilis.

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*Laboratory for Astrophysics  
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December 8, 1980

Dr. Basilis Xanthopoulos  
Department of Astronomy  
University of Thessaloniki  
Thessaloniki, Greece

Dear Basilis:

Please substitute the enclosed sheets in the nth copy for Chapter 4 and in the accompanying notes: they correct the errors about which I wrote to you on Friday.

Considerations of this kind are very tricky as I have found when doing the corresponding things for the case of Reissner-Nordstrom. Indeed the whole matter of giving a proper account of the reflexion-transmission problem in this latter case turns out to have many pitfalls. I hope that settling these issues will not postpone my schedule once again.

Sincerely,

*Chandra*

S. Chandrasekhar

SC:gw

Enclosures

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December 15, 1980

Dr. Basilis Xanthopoulos  
University of Thessaloniki  
Astronomical Department  
Thessaloniki, Greece

Dear Basilis:

Your letter of December 5 arrived this morning; and I came all ready to start the typing of Chapter 4; but my secretary could not come in today.

I am sorry that I overlooked sending pages 81-100 of the nth copy of Chapter 4. Perhaps it was a subconscious feeling that you don't care too much for the "transformation theory."

Besides the missing pages of Chapter 4, I am also including the balance of the notes for Chapter 5. There is only one additional topic I have to consider, namely, the instability of the inner horizon. I hope I can get this work done before the end of the week and start on the N-1 draft for Chapter 5 at least by early next week.

As I told you, I shall try my best to get myself prepared to come to Greece by the end of the first week of February. In all probability, we may go on to India for two weeks before returning to Chicago.

With best wishes.

Sincerely,



S. Chandrasekhar

SC:jf  
encl.

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and Space Research

November 17, 1980

Dr. Basilis Xanthopoulos  
University of Thessaloniki  
Department of Astronomy  
Thessaloniki, Greece

Dear Basilis:

During the past week I have been writing the nth copy of Chapter 4. I have made changes at various points in response to your substantive remarks, though I have not been able to accommodate all of them. However, to clarify fully your entirely justified queries in the context of your remark 15 (re: p. 82-83) and your remark 33 (re: p. 104), I find that I have to add an entirely new section clarifying why  $Y$ ,  $Z^+$  and  $Z^-$  describe the incidence, ~~of~~ reflexion, and transmission of gravitational waves. But unfortunately I cannot get around to writing it before the end of the week since I have to go to Washington later in the week for one of those assignments which I heartily dislike.

I hope you have received my proof of the diagonalization. As you can see the incorporation of this proof and some appropriate remarks on the way the metric can be generalized to allow for a  $\varphi$ -dependence will require the addition of two or three pages in Chapter 2; and what is more annoying, the renumbering of the equations!

Have you gotten your new telephone number?

With best wishes,

Yours sincerely,

  
S. Chandrasekhar

SC:gw

Enclosure



Thessaloniki, December 24, 1980.

Dear Chandra:

Yesterday I received the missing pages 81-100 of the  $n^{th}$  draft of chapter IV. And with all the formulae on hand, I've worked through the remaining of chapter IV.

Main remark: It took me quite a lot of verify the asymptotic behaviors on pages 103 and 104 and I think that the "gradient of difficulty" is much larger here than the rest of the book. Also, I am not sure if equations (365) and (366) are the definitions of  $Y_{\pm 2}^{(in)}$ ,  $Y_{\pm 2}^{(ref)}$ ,  $Y_{\pm 2}^{(tr)}$  or they can be obtained from something else. Also It took me some time to understand why the denominator in two of the expressions (361) should be  $4(i\epsilon - \frac{1}{2}m)$  instead of just  $4i\epsilon$ . What about writing the following "from the equations (319) we expect the asymptotic behavior  $Y_{+2} = A \Delta^2 e^{-i\epsilon r_x}$  with A constant and we try to evaluate A". Then, there is no way of making the mistake. Also, the introductory words of the paragraphs which starts at the end of page 102 leaves one with the impression that for  $r_x \rightarrow +\infty$ ,  $Z^{(\pm)} \rightarrow e^{+i\epsilon r_x}$  and for  $r_x \rightarrow -\infty$ ,  $Z^{(\pm)} \rightarrow e^{-i\epsilon r_x}$ , but he later realizes, for instance in equations (357) and (360) that both are needed,  $e^{\pm i\epsilon r_x}$  for  $r_x \rightarrow +\infty$  some help here might help.

Some other minor remarks:

(Page 82). Just before equation (287) you write "we shall now assume", although there is no assumption involved in writing eq. (287) for certain, to be determined, functions  $f$  and  $W$ .

(Page 86, line 6) you could write "even in  $\beta$ ".

(Page 86). Do you need to include in parenthesis  $(\frac{1}{2})^2$  in equations (305) and (306)?

(Page 107, line 11). I do not really agree that the separation of the perturbations into axial and polar is "concretized in the Newman-Penrose formalism".

$\phi_0$  and  $\psi_0$  are complex scalars which have a real and imaginary part, this is a splitting. Moreover, since we are dealing with linear equations, the real and ~~the~~ the imaginary parts should satisfy independent equations. The same remark also applies for the paragraph in the middle of page 111.

(Page 110, ending remarks of the paragraph). You conclude that, because the reflexion and transmission coefficients are the same, the solutions of any other wave equation will be expressible as linear combination of  $z^{(+)}$  and its derivative. I think that the conclusion is always true. The equal scattering should imply some regularity conditions on the coefficients of  $z^{(+)}$  and  $z^{(-)}$ .

- (Page 113, Remark after equation (383)). Since

$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx$  is finite,  $\forall t$ , you conclude that  $\Psi(x,t)$  is bounded modulo some delta function type unboundnesses. Does the spectral theory excludes these possibilities as well or there is just the conclusion expressed by equation (383)? Clarify.  
- (Page 117). I would have written  $A^{-1}(t)$  in equation (388).

Enjoy a happy New Year.  
Best regards to your wife.

Sincerely,  
Basilis.

Sorry, I've just learned that the Post office is closed today and it will be closed on the 26<sup>th</sup> as well. The letter cannot be mailed before Saturday, the 27<sup>th</sup>.

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December 5, 1980

Dr. Basilis Xanthopoulos  
University of Thessaloniki  
Department of Astronomy  
Thessaloniki, Greece

Dear Basilis:

Just a line to say that in equation (361) of Chapter 4, the formulae for  $Y_{-2}$  in the first line and  $Y_{+2}$  in the second line are in error. This error carries over to the equations on p.108. In the notes I sent, the corresponding errors are in the formulae on p.4 last line and p.5 first line; and the carried over errors on pages 6 and 7. I will correct the errors in the final typed copy. But if you wish to amuse yourself you can correct them also.

With best wishes,

Sincerely,

*Chandra*

S. Chandrasekhar

SC:gw

⊗ Essentially one has to replace  $1.0$  by  $(1.0 \pm \frac{1}{2M})$